THE CONSTRUCTING CONCEPTS OF FRACTION: REPRESENTED BY CIRCLES, RECTANGLES, AND NUMBER LINES

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Abstract

This study aims to gather information regarding the cognitive processes of 11th-grade students when constructing the concept of fractions using visual representations such as circles, rectangles, and number lines. A qualitative descriptive research approach was employed by the researcher. The data collection process began with a questionnaire consisting of 15 questions divided into five types, each type focusing on circles, rectangles, and number lines. Subsequently, students were interviewed to reinforce their answers. The collected data, comprising performance results and interviews, were analyzed to understand the students' thinking processes. The findings of this study revealed that the thinking process of 11th-grade students in understanding fractions through visual representations involved the assimilation and accommodation processes. It was observed that student S1 effectively employed assimilation in constructing the concept of fractions, while student S2 demonstrated a relatively good understanding using the accommodation process. However, student S3 struggled with the process of accommodation, resulting in a lesser degree of understanding.

Keywords: thinking process, constructing, concept of fractions, representation

1. Introduction

In learning mathematics, thinking is a very important cognitive activity. The success of learning mathematics is influenced by thinking ability. Thinking is the result of students' cognitive activities in solving problems and this can be seen through the behavior and results of student performance (Prayitno et al., 2018). There are several researchers who have examined the importance of thinking in mathematics. From the results of the studies, it was obtained the characteristics of thinking errors and the types of students' thinking processes in solving problems (Subanji & Nusantara, 2013; Subanji & Supratman, 2015). Thinking is a cognitive activity in which the activity occurs through a process. Supriadi et al (Supriadi et al., 2015) has studied the thinking process and reveals that in the thinking process there is the formation of understanding, the formation of opinions and the drawing of conclusions. The thinking process is a cognitive process that begins with receiving data, then encoding it and storing it in memory, then the data will be retrieved when the individual needs it in the next data processing (Siswono, 2007). Meanwhile, according to Prayitno & Suarniati (2017) the process of thinking is the mental activity carried out by students in solving problems and it can be seen from students' behavior which appears to be the result of completing tasks. In this study, the researcher defines the process of thinking as the course of a cognitive activity that can be observed through answers, student work, and the movements of a person in working on or solving a problem or question. In one's thinking process, there is a process between incoming new information and schemas, thus the incoming information will be adjusted to the assimilation and accommodation process (Simatwa, 2010; Subanji & Supratman, 2015).

The assimilation process is a cognitive process in which a person combines or integrates new perceptions, concepts or experiences into the schemes or pattern in his mind. While, the accommodation process refers to a process in which a person combines or unifies new stimuli through changing old existing schemes (internal structures of knowledge) to adapt to existing problems (Subanji & Supratman, 2015). According to Piaget, the assimilation and accommodation process the ways through which children integrate new experiences into already existing cognition structures (schema) and later will form a new scheme (Gembong, 2016). In this case, when someone is in the process of learning to gain an understanding of a knowledge, he will try to renew or alter his existing understanding or experience with the new one by assimilating into existing schemes or reconstructing these schemes to accommodate this knowledge (Hackenberg, 2010).

Mathematics learning is heavily influenced by the philosophy of constructivism which emphasizes that knowledge is formed (constructed) from oneself. Several researchers have examined topic related to the term 'construction' and on how individuals can construct the concept. From the results of this study, the researcher explained that information processing theory is used to construct a concept or knowledge (Subanji, 2017). Information processing theory explains the construction of knowledge, from entering information, filtering, processing, storing, to recalling or retrieving information in knowledge storage. A lot of information (in the form of external stimulus) show up every time and is selected through sensory memory. Unimportant information will be ignored (forgotten), while important information will be continued to the short-term memory and being processed by utilizing (calling up) information in the long-term memory. There are 4 (four) results of information processing, the continuation of short-term memory, namely: (1) the results of processing that are not important will be forgotten, (2) the results of processing that are very important and have not been completed in processing will be repeated, (3) the results of processing that are action is needed, a response will appear, and (4) the results of information processing that have been completed are coded and stored in long-term memory.

In the learning application, fractions require a deeper understanding. This understanding is obtained from learning whole numbers, because fractions are numbers that can be represented by a pair of whole numbers $\frac{a}{b}$, where b ≠ 0 (Musser et al., 2013; Siegler et al., 2013). Learning and understanding related to the concept of fractions has started since elementary school. However, students do not really understand the concept of fractions because they are still in the first stage of learning, namely memorization. This can be seen when students mentioned the concept of fractions symbolically, yet they were unable to express the meaning of the symbols (Permadi & Irawan, 2016). It can be said that learning fractions is difficult for children in general. In line with Tunç-Pekkan statement (2015) that fractions are perceived as one of the most difficult lessons in mathematics to learn in all countries. As can be seen, the results of the US National Assessment of Educational Progress (National Assessment of Educational Progress (NAEP), 2007) study reported that 60 % of 4thgradestudents could not determine whether $\frac{1}{4}$ was greater than $\frac{1}{5}$, and half of 8th-grade students did not choose the correct option when ordering three fractions from least to greatest. In addition, the TIMSS Numeracy 2015 results (which are only related to the concept of fractions) showed that Indonesian students' ability regarding the concept of fractions is relatively low because only 42.67 % of students answered correctly comparing to the international level, namely 47.33 %. The results of Indonesian students are lower than the percentage of students' correct answers in countries with a TIMSS score below Indonesia, namely Jordan (TIMSS score of 388) of 46.7 % and South Africa (TIMSS score of 376) of 48.72 %.

There are several factors that make fraction so difficult concepts to learn, namely the sub-constructs (Pantziara & Philippou, 2012). Sub-constructs refer to knowledge pieces to understand how a fraction problem can be solved (Tunç-Pekkan, 2015). To construct the concept of fractions, there are five interrelated sub-constructs, namely part-whole, ratio, measure, operator, and quotient (Charalambous & Pitta-Pantazi, 2007). In this study, researchers focused on two subconstructs to find out students' thinking processes, namely the part-whole and measure subconstructs. The part-whole sub-construct can be used to identify units and common fractions while the measure sub-construct can be used to construct ordinary fractions, reconstruct units from their parts or mixed fractions, and create and identify the number of mixed fractions (Tunç-Pekkan, 2015).

In forming the fractional scheme and its operations, (Hackenberg, 2013; Izsak et al., 2008) summarized into 5 schemes, namely 1) Parts-within-wholes fraction scheme in which there is only partitioning operations observed and the result of the parts might not be equal. This scheme is used to identify fraction symbols for the already partitioned and shaded drawings of pie charts, without attending to the parts being equal. 2) part-whole fraction scheme, where this scheme is used to arrange fractions by dividing the whole into equal parts. Then taking a part out of a whole without mentally destroying the whole to make up the fraction. 3) Partitive unit fraction schemes, this scheme is used to determine the units of the unit fractions that have been given, by partitioning the shape into some equal parts, then takes one of the partitions, and iterates it to make the whole. 4) Partitive fractional scheme, this scheme is used to determine the genuine fraction of the whole that has not been divided. By dividing the genuine fraction to get the units fraction. It then repeats the unit fraction to create the genuine fraction and the whole. Repetition in this scheme is only limited to all parts. 5) iterative fractional scheme, this scheme is used to complete the formation of ordinary fractions and mixed fractions by dividing the same parts so as to produce unit fractions and then repeating them to determine the whole and units of the fraction.

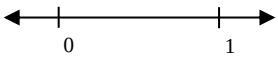
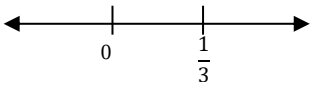
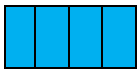
Several researchers have studied fractions, including Pekkan (2015b) who studied elementary school students' knowledge of fractions represented in circles, rectangles and number diagrams. Six-hundred and fifty-six 4th and 5th grade students took the test on Pekkan study (2015b) and were given six fractional Problem Types (a total of 18 questions). The findings of this study indicated that students showed similar performance in circle and rectangle items that required using part-whole fractional reasoning, but students’ performance was significantly lower on the items with number line graphical representation across the Problem Types used. Pekkan's study (2015b) focused his research on students' knowledge of fractions. However, one limitation on research conducted by Pekkan (2015b) was that "... further research is needed for understanding children's construction of (especially) the number line representation in the fractional knowledge context". This means that it is still necessary for research related to constructing the concept of fractions represented via a number line. Pekkan's research (2015b) used quantitative research methods which only explained the percentage of students' knowledge related to fractions without explaining the causes of the problem. Therefore, qualitative research methods were used in this study.

Based on the description above, it can be seen that learning the concept of fractions is a difficult lesson and requires more understanding and further studies are still needed related to Pekkan's research (2015b). Therefore, describing the thinking processes of class XI students in constructing the concept of fractions represented by circles, rectangles and number lines becomes the aim of this study.

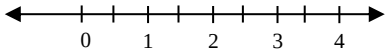
2. Research Method

To be able to answer this research objective, this research uses a qualitative descriptive research approach. The subjects in this study were 3 students of class XI, in which the subject was taken based on the subject's willingness to join the research and had learned the concept of fractions. The instrument in this study was adopted from the Pekkan (2015b), namely An Analysis of Elementary School Children's Fractional Knowledge Depicted with Circle, Rectangle, and Number Line Representations. This research instrument only took four of the six types of questions including the arrangement of fractions, the reconstruction the unit from unit fractions items, the naming of mixed fractions, and the reconstruction of units from mixed fractions items. Each of these types of questions has 3 questions items which include representations of circles, rectangles and number lines. The questions given to students are as follows:

Table 1. Research Instruments

Questions	Answer	Explanation
If this  circle is the unit, Please draw a picture to show $\frac{3}{4}$ of this unit.		
If this rectangle is the unit,  Please draw a picture to show $\frac{5}{6}$ of this unit.		
Place $\frac{2}{3}$ on the number line.  		
 If this picture is $\frac{1}{4}$ of an amout, please draw the picture of that amount. (It does not have to be the full circle)		
If this picture is $\frac{1}{5}$ of an amount.  please draw the picture of that amount.		
Place “1” on the number line.  		
 If this circle is the unit, what would be the fraction shown by the shaded parts of the diagrams below? 		
If Rectangle A (upper rectangle) is the unit, what fraction does Rectangle B (bottom rectangle) show? A  B 		

Place $\frac{3}{2}$ on the number line



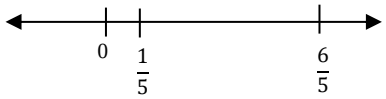
If this circle shows $\frac{5}{3}$.

Place draw the unit.

If this rectangle shows $\frac{5}{4}$.

Please show the unit.

Place "1" on the number line



To collect the data, the researchers administered research instruments in the form of questions to 3 class XI students which are completed within 30 minutes. While writing answers, the three students were interviewed to strengthen or clarify their answers. Then the results of students work and interviews are used to describe and explain students' thought processes in constructing the concept of fractions which are represented by circles, rectangles, and number lines. The qualitative data that has been obtained will be analyzed using the stages of analysis developed by Creswell (2009).

3. Result And Discussion

Result

The data described is the result of student work. The following is the result of student work in solving the 12 questions given.

Composing Fractions

The thinking process of S1, S2, and S3 in composing fractional numbers which are represented by circles, rectangles, and number lines begins by reading the questions that have been given. After reading the questions, students understand the problem and retrieve the concept of composing fractions that has been studied before. The following is the result of S1's work.

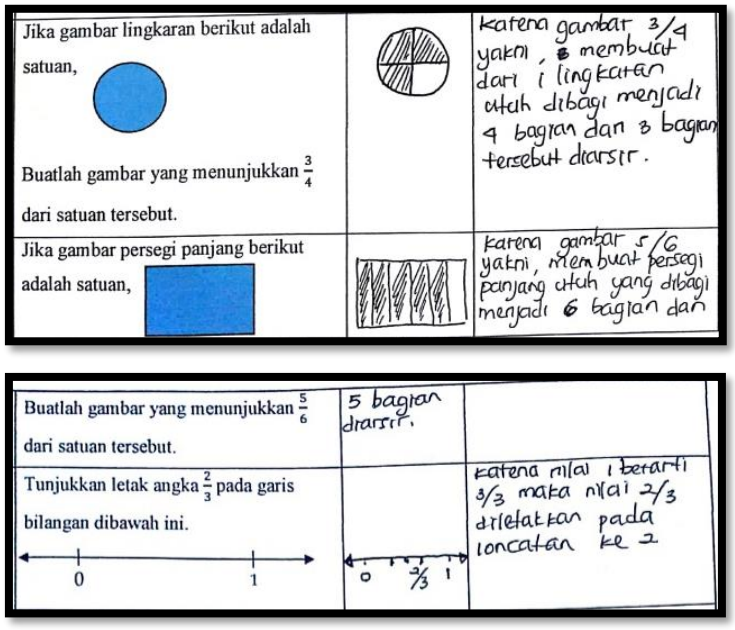


Figure 1. The results of S1's task

Based on the results of S1's task, it can be seen that S1 correctly performed the unit problems by giving a fraction symbol for the shaded representation for circle and rectangle problems respectively. For questions that were represented by circles, S1 described a circle by dividing into 4 equal parts where 3 parts are shaded to shows the numerator and the sum of all the parts is the denominator. Not much different from the previous problem, for the rectangle item, he described a rectangle with the same 6 parts where 5 of the parts are shaded as the previous problem. Whereas for the number line item, a fraction symbol was asked for the indicated point on a number line. S1 was putting $\frac{2}{3}$ between $\frac{1}{3}$ and 1 where the 0–1 interval was partitioned equally with marks. Here are the results of S2 task as follows.

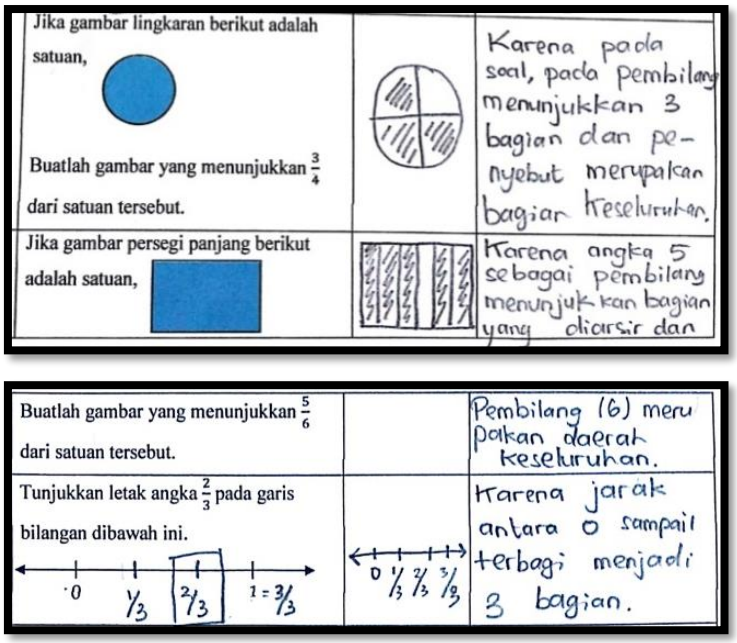


Figure 2. The results of S2's task

Based on the results of S2's task, it can be seen that S2 correctly performed the unit problems by giving a fraction symbol for the shaded representation for circle and rectangle problems respectively. For questions that were represented by circles, S2 described a circle by dividing into 4 equal parts where 3 parts are shaded to shows the numerator and the sum of all the parts is the denominator. Not much different from the previous problem, for the rectangle item, he described a rectangle with the same 6 parts where 5 of the parts are shaded as the previous problem. Whereas for the number line item, a fraction symbol was asked for the indicated point on a number line. S2 was putting $\frac{2}{3}$ between $\frac{1}{3}$ and 1 where the 0–1 interval was partitioned equally with marks, then sorting them up to $\frac{2}{3}$. Here are the results of S3 task as follows.

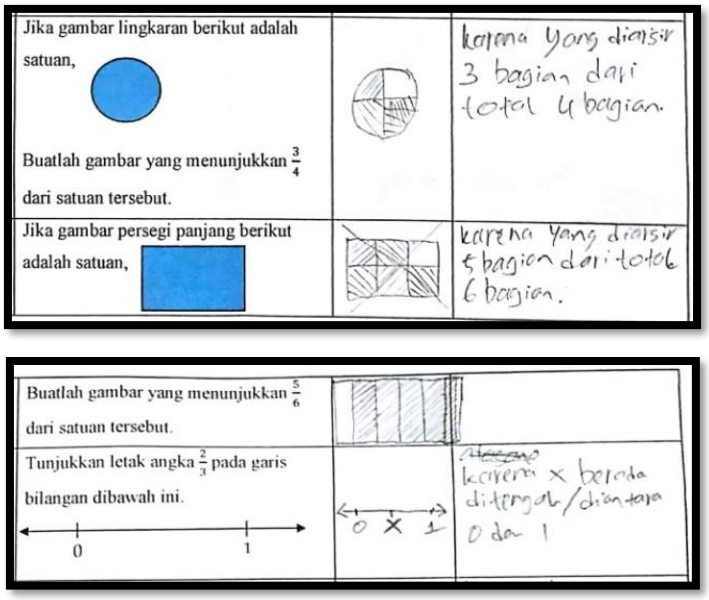


Figure 3. Results of S3 Task

Based on the results of S3's task, it can be seen that S3 correctly performed the unit problems by giving a fraction symbol for the shaded representation for circle and rectangle problems respectively. For questions that were represented by circles, S3 described a circle by dividing into 4 equal parts where 3 parts are shaded to shows the numerator and the sum of all the parts is the denominator. Not much different from the previous problem, for the rectangle item, he described a rectangle with the same 6 parts where 5 of the parts are shaded as the previous problem. Whereas for the number line item, a fraction symbol was asked for the indicated point on a number line. S3 was putting $\frac{2}{3}$ between 0 and 1 where the 0–1 interval was partitioned equally with 2 equal marks.

Reconstruction the Unit from Unit Fractions Items

The thinking process of S1, S2, and S3 in reconstructing units from unit fractions represented by circles, rectangles, and number lines. First, the students were reading the questions. After reading the questions given, students understood the problem and retrieve the concept of compiling fractions that has been studied before. The following is the result of S1's task.

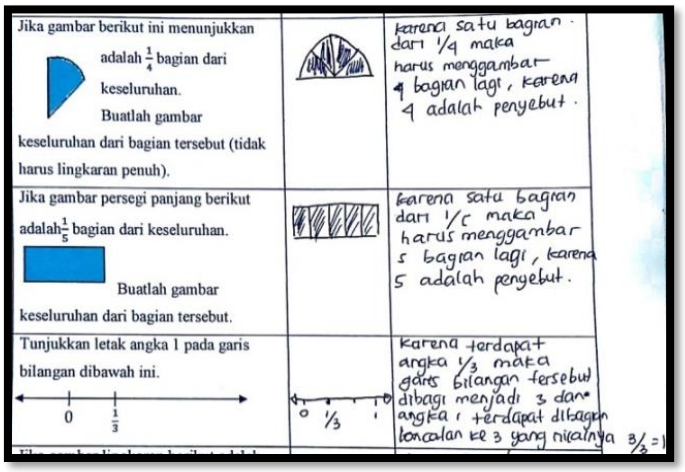


Figure 4. The results of S1's task

Based on the results of the student's task, it can be seen that for questions that are represented by circles. S1 described the problem by reversing their thinking such that a part is

given and they are to draw a whole. S1 performed well on both circle and rectangle items by drawing half circle for the whole because by repeating $\frac{1}{4}$ of the existing parts, it became a half circle with 4 parts in total. For rectangle item, S1 did similar attempt. S1 described a rectangle with the same 5 parts where all the parts are shaded for the same reasons as before. Whereas for questions that are represented by a number line, S1 put 1 after $\frac{2}{3}$ where the 0–1 interval was partitioned equally with 3 equal marks. Meanwhile, the results of S2 work are as follows.

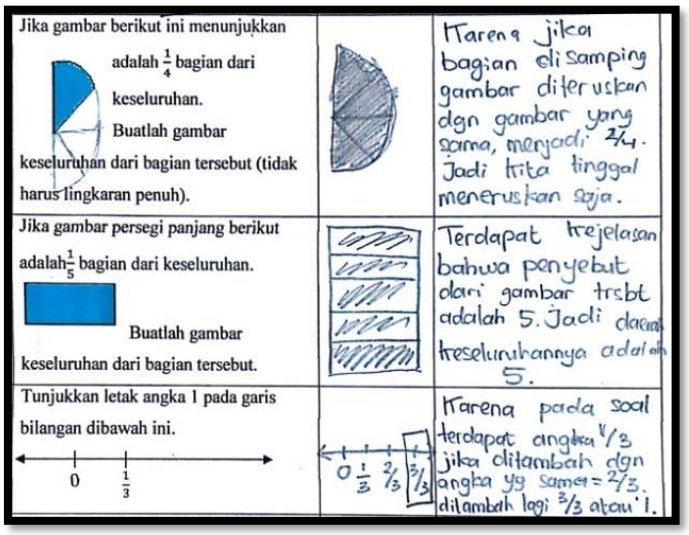


Figure 5. The results of S2's task

Based on the results of the student's task, it can be seen that for questions that are represented by circles. S2 described the problem by reversing their thinking such that a part is given and they are to draw a whole. S2 performed well on both circle and rectangle items by drawing half circle for the whole because by repeating $\frac{1}{4}$ of the existing parts, it became a half circle with 4 parts in total. For rectangle item, S1 did similar attempt. S2 described a rectangle with the same 5 parts where all the parts are shaded for the same reasons as before. Whereas for questions that are represented by a number line, S2 put 1 after $\frac{2}{3}$ by repeating the numbers $\frac{1}{3}$ to $\frac{3}{3} = 1$. Then, for the results of S3 work are as follows.

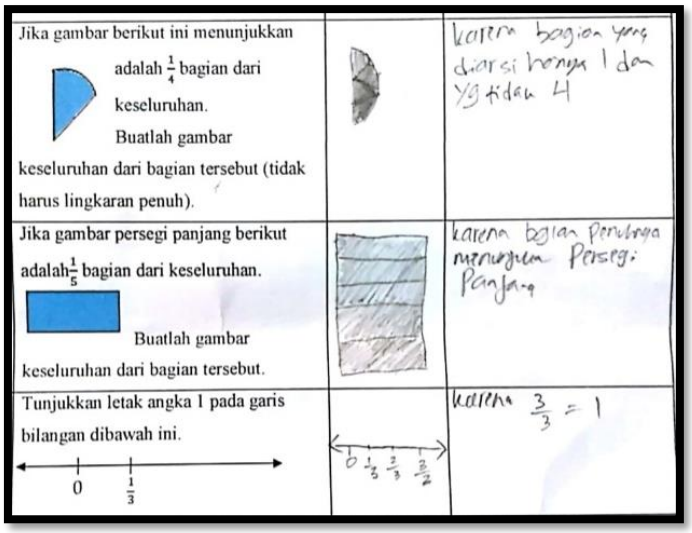


Figure 6. The results of S3's task

Based on the results of the student's task, it can be seen that for questions that are represented by circles. S3 described the problem by reversing their thinking such that a part is given and they are to draw a whole. S3 performed well on both circle and rectangle items by drawing half circle for the whole because by repeating $\frac{1}{4}$ of the existing parts, it became a half circle with 4 parts in total. For rectangle item, S3 did similar attempt. S3 described a rectangle with the same 5 parts where all the parts are shaded for the same reasons as before. Whereas for questions that are represented by a number line, S1 put 1 after $\frac{2}{3}$ by repeating the numbers $\frac{1}{3}$ to $\frac{3}{3} = 1$.

Mixed Fraction Naming

The thinking process of S1, S2, and S3 in naming mixed fractions represented by circles, rectangles, and number lines. First, the students were reading the questions. After reading the questions given, students understood the problem and retrieve the concept of compiling fractions that has been studied before. The following is the result of S1's task.

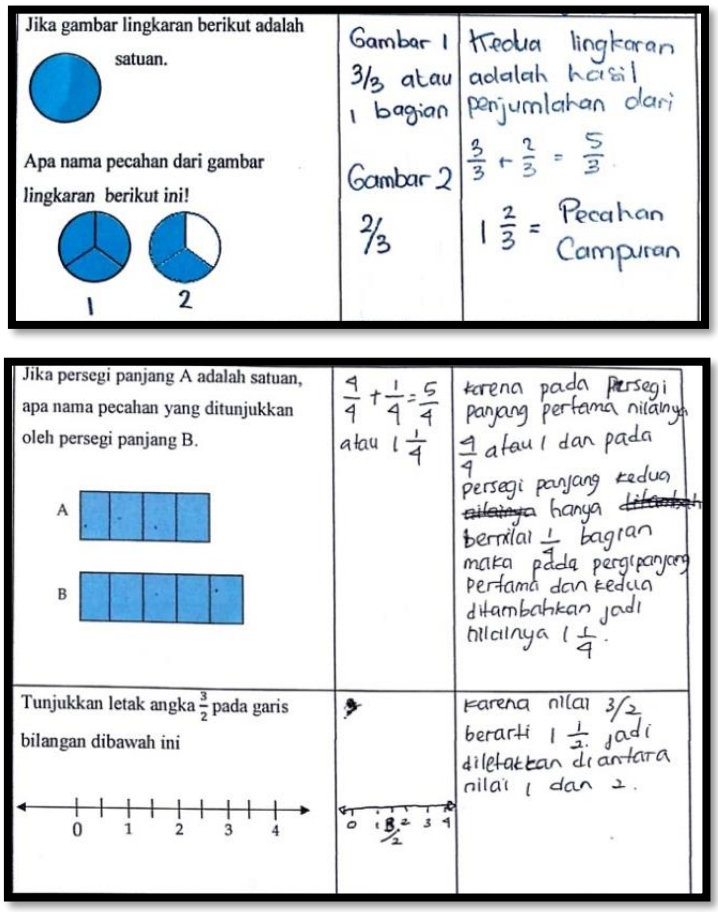


Figure 7. The results of S1's task

Based on the results of the student's task, it can be seen that for questions represented by the circle, S1 wrote $1\frac{2}{3}$ as the answer by adding up the two circle values. For questions that are represented by rectangles, S1 did it by using the same way, that is S1 wrote $1\frac{1}{4}$ for the same

explanation as previous problem. As for the questions represented by the number line, S1 put $\frac{3}{2}$ between 1 and 2 by changing $\frac{3}{2}$ to $1\frac{1}{2}$. Meanwhile, the results of S2 task are shown as follows.

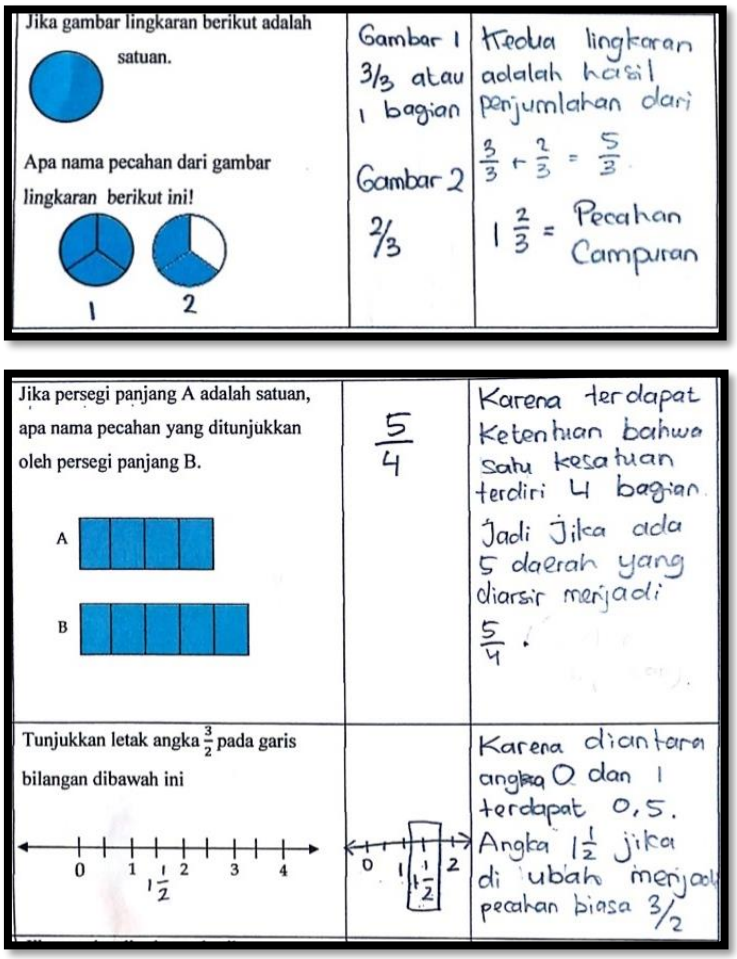


Figure 8. The results of S2's task

Based on the results of the student's task, it can be seen that for questions represented by the circle, S2 wrote $1\frac{2}{3}$ as the answer by adding up the two circle values. For questions that are represented by rectangles, S2 wrote $\frac{5}{4}$ and explained that there were four parts on the first rectangle and five shaded parts on the second rectangle. S2 put $\frac{3}{2}$ between 1 and 2 by changing $\frac{3}{2}$ to $1\frac{1}{2}$. And for the results of S3 task are presented as follows.

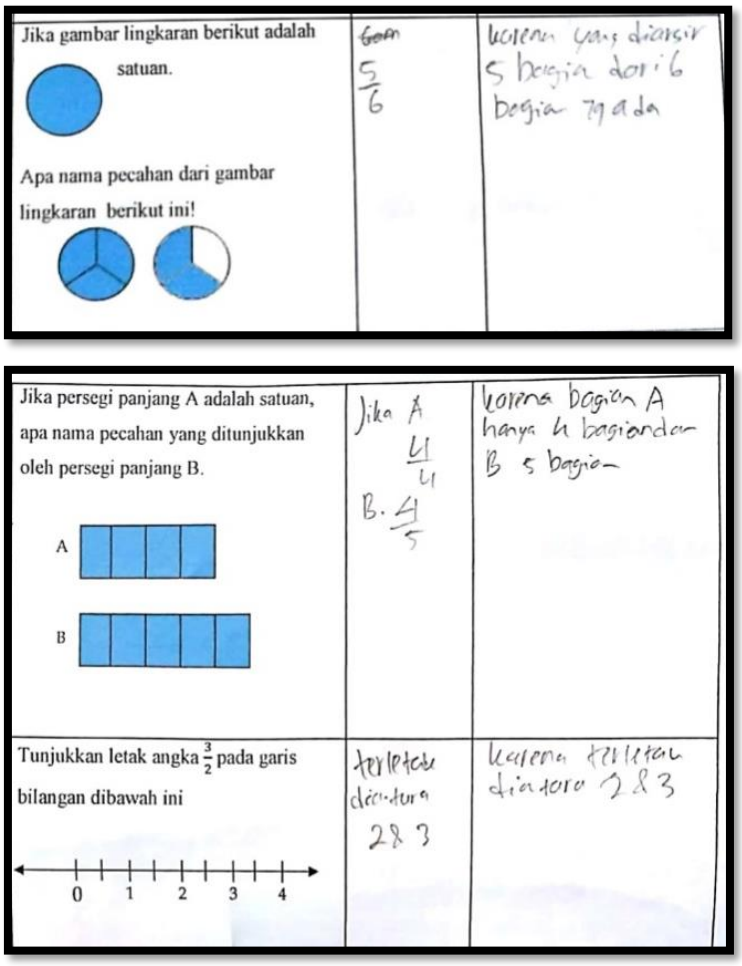


Figure 9. The results of S3's task

Based on the results of the student's task, it can be seen that for questions represented by the circle, S3 wrote $\frac{5}{6}$ as the answer because there are five parts that are shaded out of the six parts. For questions that are represented by rectangles, S3 did it by using the same way, that is S3 wrote $\frac{4}{5}$ because there are four parts in the first rectangle and five shaded parts in the second rectangle. As for the questions represented by the number line, S3 put $\frac{3}{2}$ between 2 and 3.

Reconstruction of units from mixed fractions

The thinking process of S1, S2, and S3 in reconstructing units from mixed fractions represented by circles, rectangles, and number lines. First, the students were reading the questions. After reading the questions given, students understood the problem and retrieve the concept of compiling fractions that has been studied before. The following is the result of S1's task.

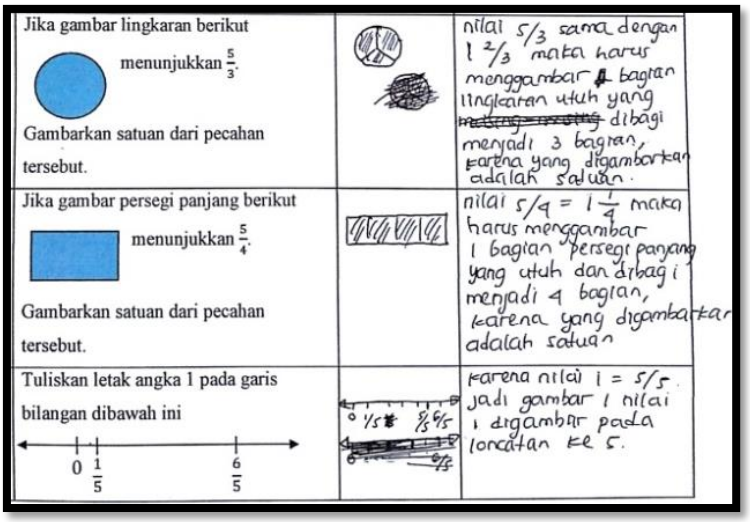


Figure 10. The results of S1's task

Based on the results of the student's task, it can be seen that for questions represented by the circle, S1described a circle with three equal parts and all the parts are shaded because when it is drawn, it shows the whole. There was no significant difference of the answer between circle and rectangle. For the problem that is represented by a rectangle, S1 described a rectangle with 4 equal and shaded parts for the same reason as circle question. As for the questions represented by the number line, S1 put 1 between $\frac{4}{5}$ and $\frac{6}{5}$ by sorting from $\frac{1}{5}$ to $\frac{5}{5} = 1$. While the results of the work of S2 are as follows.

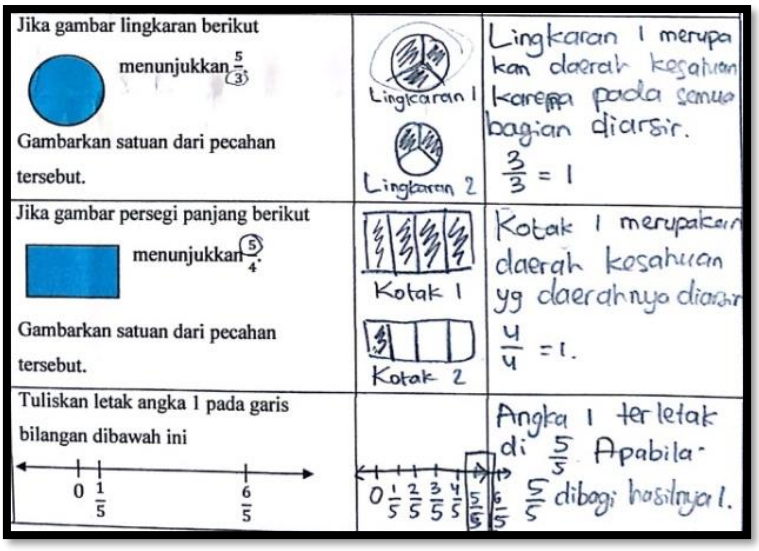


Figure 11. The results of S2's task

Based on the results of the student's task, it can be seen that for questions represented by the circle, S2 described a circle with three equal parts and all the parts are shaded because when it is drawn, it shows the whole. There was no significant difference of the answer between circle and rectangle. For the problem that is represented by a rectangle, S2 described a rectangle with 4 equal and shaded parts for the same reason as circle question. As for the questions represented by the number line, S2 put 1 between $\frac{4}{5}$ and $\frac{6}{5}$ by sorting from $\frac{1}{5}$ to $\frac{5}{5} = 1$. While the results of the work of S3 are as follows.

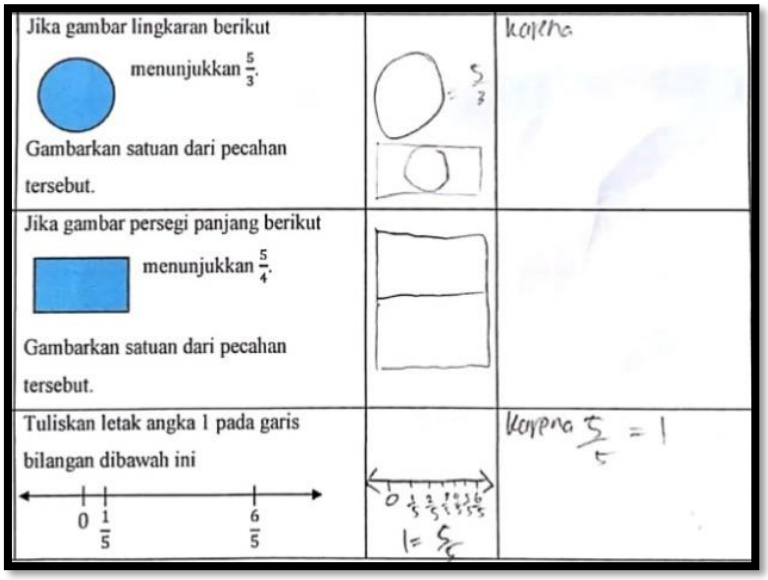


Figure 12. The results of S3's task

Based on the results of the student's task, it can be seen that for questions represented by the circle, S3 described a circle with three equal parts and all the parts are shaded because when it is drawn, it shows the whole. There was no significant difference of the answer between circle and rectangle. For the problem that is represented by a rectangle, S3 described a rectangle with 4 equal and shaded parts for the same reason as circle question. As for the questions represented by the number line, S3 put 1 between $\frac{4}{5}$ and $\frac{6}{5}$ by sorting from $\frac{1}{5}$ to $\frac{5}{5} = 1$.

Discussion

Formulation of Fractional Numbers

S1 Thinking Process in Composing Fractional Numbers Represented by Circles, Rectangle, and Number Lines

Based on the data exposure, the results are obtained. The results showed that S1 underwent an assimilation process in the formation of forming understanding on all types of questions, in which understanding the questions without a process, or being able to understand by reading the questions given directly (Kurniawan et al., 2017). The selected information is then delivered to the short-term memory and processed in the long-term memory to link or combine the newly entered information with the existing schema. In this case, S1 used a part-whole scheme, where this scheme is used to arrange fractions by dividing the whole into equal parts. Then, the whole part is taken without changing the whole to compose the fraction (Hackenberg, 2013; Tunç-Pekkan, 2015). In the process of forming this understanding, S1 undergoes an assimilation process, in which the assimilation process is associating or incorporating new information into the schema he already has (Subanji & Supratman, 2015).

In contrast to the circle and rectangular representations, in this number line representation S1 did not use a part-whole fraction scheme, but S1 used the scheme described by H. Wu (2011) that to draw fractions on the number line, namely (1) describes the fraction $\frac{1}{b}$ on the number line by plotting the interval from 0 to 1 as a unit and dividing it into b equal parts. Recognize that each part has a size of $\frac{1}{b}$ and the end point (defined point) of the part at 0 places the number $\frac{1}{b}$ on the number line; (2) describes the fraction $\frac{a}{b}$ on the number line by marking a length $\frac{1}{b}$ from 0. So that in the process of drawing this conclusion, S1 proceed an assimilation in which S1 immediately integrates the new information into the old scheme (Rizal, 2011).

S2 Thinking Process in Composing Fractional Numbers Represented by Circles, Rectangle, and Number Lines

S2 attains well in his thinking process in composing fractions which are represented by circles, rectangles and number lines. It can be seen when S2 understands the problem well without repeating it, so that in forming this understanding S2 experiences an assimilation process (Kurniawan et al., 2017). In the process of forming ideas, S2 experiences a process of assimilation, in which S2 integrates new information with the schema she/he has (Subanji & Supratman, 2015). In compiling these fractions, S2 forms a part-whole fraction scheme. This is in line with the statements of Hackenberg and Pekkan (2013; 2015b) regarding the fractional scheme which explains that to arrange fractions by dividing the whole into equal parts. Then, taking a part out of a whole without mentally destroying the whole operation to compose the fraction. In drawing conclusions which are represented by circles, S2 sketches or tries to draw first on the problem. In this case, S2 does not directly do on the fraction problem or requires a process to arrange fractions. Thus, S2 experiences an accommodation process. Kurniawan (2017) reported that when students need a process or indirectly in solving mathematic problem, they experience an accommodation process. As for the rectangular and number line representations, S2 undergoes an assimilation process.

S3 Thinking Process in Composing Fractional Numbers Represented by Circles, Rectangle, and Number Lines

S3 attains well in his thinking process in composing fractions which are represented by circles, rectangles and number lines. It can be seen when S3 could understand the problem without repeating in reading of the given problem, so that in forming this understanding S3 goes through an assimilation process (Kurniawan et al., 2017). The information obtained is continued into the short-term memory and then processed in the long-term memory, so that the related information will be retrieved and matched again with the newly obtained information. In this case, S3 used a fractional scheme which is contrary to Hackenberg and Pekkan (2013; 2015b) so that in this case S3 experienced an accommodation process in which S3 combined or unified new stimuli through changing the old scheme to adapt to existing problems (Subanji & Supratman, 2015). S3 did not believe in the scheme she/he already had so that in forming opinions, S3experienced process of accommodation (Rizal, 2011). Further, in drawing conclusions, S3 underwent an assimilation process in the circle representation. Whereas for the rectangular representation S3 experienced an accommodation process seen when S3 replaced the previous answer (Kurniawan et al., 2017).

Meanwhile, S3 could not attain satisfactory result in the preparation of fractional numbers represented by the number line. It was seen when S3 was able to determine the information used to solve the problems given in the process of forming understanding. However, she/he was not able to determine the steps in solving it through the process of forming opinions, so that in this process S3 experienced an accommodation process (Kurniawan et al., 2017). Because she/he was unable to determine the steps at the previous stage resulting in errors in answering, imperfections occurred in the process of drawing conclusions by S3.

Unit Reconstruction from Unit Fractions

S1's thinking process in reconstructing units from unit fractions represented by circles, rectangles, and number lines (S1)

S1's thinking process in reconstructing units from unit fractions represented by circles, rectangles, and number lines is going well. It can be seen when S1 can understand and find the information needed to solve the problem directly without doing repetition when reading the problem. So that in the process of forming this understanding S1 underwent an assimilation process (Kurniawan et al., 2017; Rizal, 2011). The information obtained is the result of selection

which is then entered into the short-term memory and will be processed in the long-term memory by retrieving the information held with the information obtained. From this process S1 stated that to determine the units of unit fractions using the division scheme of unit fractions, S1 repeated the unit fractions to produce the whole (Izsak et al., 2008). S1 experienced an assimilation process in forming this opinion, it was seen when S1 integrated the new information with the schema she/he had (Subanji & Supratman, 2015). As for drawing conclusions, S1 is able to use information, steps, and concepts that have been planned to solve the problem, so that in this case S1 experiences an assimilation process.

S2's thinking process in reconstructing units from unit fractions represented by circles, rectangles, and number lines (S2)

S2's thinking process in reconstructing units from unit fractions represented by circles, rectangles, and number lines goes well. It can be seen when S2 can understand and determine the information used without repeating when reading the questions, so that in the formation of this understanding S2 experiences a process of assimilation (Sahlberg, 2020a). S2 stated that to reconstruct units from unit fractions, S2 used a division scheme of unit fractions by repeating the unit fractions to produce the whole (Stevens et al., 2020). So that in this case S2 experiences an assimilation process. It can be verified when S2 integrated incoming information with the schema she/he had (Subanji & Supratman, 2015). As for the process of drawing conclusions S2 required a process to use the information and schemes that are already owned. S2 first sketched his/her answer on the picture shown in the question, so in this case S2 experienced an accommodation process. Unlike the circle representation, in the rectangle and number line S2 wrote the answer directly without sketching it first. So that in this case S2 experienced an assimilation process.

S3 thinking process in reconstructing units from unit fractions represented by circles, rectangles, and number lines (S3)

S3's thinking process in reconstructing units from unit fractions represented by circles, rectangles, and number lines is not going well. It can be seen when S3 did repetition in reading the questions to understand them and got information related to reconstructing units from unit fractions. S3 also experienced imperfections in formulating his/her understanding in which S3 lacked confidence that she/he had been obtained sufficient information to solve the problem, so it can be concluded that S3 experienced an accommodation process (Steffe, 2001). Then the information obtained is the result of the selection and will be entered in the short-term memory and processed in the long-term memory. The information in the long term memory is retrieved and matched with the new information.

In this case, S3 stated that to reconstruct units from unit fractions was used the division scheme of unit fractions, by repeating the unit fractions to produce the whole (Hackenberg & Tillema, 2009). So that in forming opinions S3 experienced an accommodation process, which is able to determine the relationship between the information obtained and the schema that is owned, but in this process S3 required a process to be able to retrieve the scheme from he/she long-term memory. In the process of drawing conclusions, S3 experienced an accommodation process, as seen when S3 was not perfect in using its scheme. It can be seen when S3 feels unsure that the scheme is a concept for solving existing problems.

Mixed Fraction Naming

S1's thinking process in naming mixed fractions represented by circles, rectangles and number lines (S1)

S1's thinking process in naming mixed fractions which are represented by circles, rectangles and number lines goes well. The process began with reading the questions, so from reading the questions S1 got information to solve the questions and what was asked of the questions. In the process of forming this understanding S1 underwent a process of assimilation

(Stevens et al., 2020). Based on the researcher's interview with S1 regarding the scheme used to solve the problem, S1 could explain the relationship between the information obtained and the scheme it had. In this case S1 modified the scheme he already had to solve the problem, so that in the process of forming this opinion S1 experienced a process of accommodation, which combined or unified new stimuli through changing the old scheme to adapt to existing problems (Subanji & Supratman, 2015). In solving these problems, S1 used the concepts that have been planned beforehand correctly and was able to check again and felt confident about the steps used to solve the problem. As the result, in the process of forming these conclusions S1 underwent a process of assimilation.

S2's thinking process in naming mixed fractions represented by circles, rectangles and number lines (S2)

S2's thinking process in naming mixed fractions represented by circles, rectangles, and number lines goes well. The process began with receiving information through reading the questions given. From this process S2 could find information that can be used to solve the problem, so that in the process of forming this understanding S2 experienced an assimilation process (Stevens et al., 2020). Based on the researcher's interview with S2 regarding the scheme used to solve the problem, S2 could explain the scheme he/she had by modifying it even though in fact, it was not correct. From this scheme S2 related it to the information that has been obtained to solve the problem. So that in forming these opinions S2 experienced a process of accommodation, which combines or unites new stimuli through changing old schemes to adapt to existing problems (Subanji & Supratman, 2015).

In solving these questions S2 used the concepts previously described and went well. However, in the process of solving the problem, which was represented by a circle and a number line, S2 sketched the answer first on the image shown in the question, so that in this case S2 experienced an accommodation process. Unlike the questions that were represented by rectangles, S2 immediately wrote down the answers without sketching them first. So that in this case S2 experienced an assimilation process.

S3 thinking process in naming mixed fractions represented by circles, rectangles and number lines (S3)

S3's thinking process in naming mixed fractions represented by circles, rectangles, and number lines is not going well. During the process of forming understanding, S3 experienced an accommodation process, where it was less able to understand the questions and it took time to get the information used to solve the questions (Subanji & Supratman, 2015). The information obtained is the result of the selection which is then entered in the short-term memory and processed in the long-term memory. In the formation process opinion, it was not perfect, it was seen when S3 was unable to connect the information obtained with the scheme he owned. So that in this case S3 experienced an accommodation process which combines or unites new stimuli through changing the old scheme to adapt to existing problems (Subanji & Supratman, 2015). In the process of drawing conclusions, S3 experienced an accommodation process seen when S3 was unable to carry out the problem-solving plan, due to the subject's error when determining alternative error steps in the previous stage.

Unit Reconstruction from Mixed Fractions

S1's thinking process in reconstructing units from mixed fractions represented by circles, rectangles, and number lines (S1)

S1's thinking process in reconstructing units from mixed fractions represented by circles, rectangles, and number lines is not going well. The process began with reading the questions given resulting in S1 receiving information. In identifying the questions that are represented by the S1 circle, S1 required a process of reading the questions repeatedly until he/she got the information he/she needed. This is in line with Yovan's statement (2008) which stated that

repetition in reading questions can improve information recall due to strengthening the relationship between information held. So that in this process S1 experienced an accommodation process (El-Nakhel et al., 2019). Whereas for questions that are represented by rectangles and number lines, S1 could determine the information used in solving the problem and what was asked in the problem properly without repeating in reading the problem. So that in the process of forming this understanding S1 went through a process of assimilation.

The information obtained is then processed in long-term memory by retrieving information related to reconstructing units from mixed fractions. In solving these questions, S1 modified his scheme to suit the questions given, so that in the process of forming opinions, S1 experienced a process of accommodation (Subanji & Supratman, 2015). In solving these problems S1 used the concepts that have been explained before well. However, for questions that are represented by circles, S1 lacked confidence in the steps and answers, so S1 checked again and then confirmed the answers that have been written. So that in the process of drawing conclusions, S1 experienced a process of accommodation. Meanwhile, for questions that are represented by rectangles and number lines, S1 believed in the steps used to solve the problem. In this process S1 experienced a process of assimilation.

S2's thinking process in reconstructing units from mixed fractions represented by circles, rectangles, and number lines (S2)

S2's thinking process in reconstructing units from mixed fractions represented by circles, rectangles, and number lines goes well. The process began with reading the questions given resulting in S2 receiving information. S2 could determine the information used in solving the problem and what was asked in the problem properly without repeating in reading the problem. As a result, in the process of forming this understanding S2 went through a process of assimilation (Hackenberg & Tillema, 2009). The information obtained is then processed in long-term memory by retrieving information related to reconstructing units from mixed fractions. In solving these questions S2 modified his scheme to suit the questions given, so that in the process of forming opinions S2 experienced a process of accommodation (Subanji & Supratman, 2015). In solving these problems S2 used the concepts that have been explained previously well. S2 believed in the steps used to solve the problem. As a result, in the process of drawing conclusions, S2 underwent a process of assimilation (Subanji & Supratman, 2015).

S3 thinking process in reconstructing units from mixed fractions represented by circles, rectangles, and number lines (S3)

S3's thinking process in reconstructing units from mixed fractions represented by circles, rectangles, and number lines is not good. The process began with reading the questions given, resulting in S3 receiving information. In the process of forming understanding in the problem, reconstructing units represented by circles, rectangles, and number lines, S3 could not immediately identify the problem properly, which still required a process to be able to find information to solve the problem. In forming this understanding S3 was experiencing an accommodation process (Hackenberg & Tillema, 2009). From this process, S3 could not find any information related to reconstructing units from mixed fractions represented by circles and rectangles. However, it could determine the information on the problem that is represented by a number line.

Based on the researcher's interview with S3, it can be seen that S3 could not state what schemes could be used to solve problems that are represented by circles and rectangles. There are many factors that cause students to be unable to solve the problem properly, namely due to their limited experience or knowledge. In addition, the concepts or knowledge used to respond to stimuli are not stored properly (Hidayati, 2021). However, unlike the questions that are represented by a number line, S3 could explain a scheme used to solve problems even though she/he still requires processing. The process of forming opinions in S3 can be seen when S3

could determine the relationship between the information it has and the existing schemes in its long-term memory. In this case S3 modified the scheme that he already has by matching the new information with the scheme he/she had, so that in the process of forming the opinion S3 experiences a process of accommodation (Sahlberg, 2020b). When S3 could not determine information that can be used to solve questions that are represented by circles and rectangles in the process of forming opinions so that in the process of drawing conclusions S3 could not answer these questions correctly. This is because the information needed in short-term memory is not properly stored in long-term memory. As for the problems represented by the S3 number line, she/he solved it using a pre-planned scheme. In the process of drawing conclusions, S3 experienced an accommodation process, it is proven when S3 did not feel confident about the steps he/she was taking to solve the problem.

4. Conclusion

The thinking process of each student in constructing the concept of fractions which are represented by circles, rectangles and number lines varies according to the type of problem given, so that the following conclusions are obtained:

The thinking process of student 1 (S1) in constructing the concept of fractions which are represented by circles, rectangles, and number lines is going well. in the process of forming the understanding, S1 can immediately understand which questions without repeating a lot in reading the questions. For the process of forming opinions, in this case, S1 uses the scheme he/she has to solve the questions given. As for drawing conclusions, S1 uses the plan he has to solve questions confidently and correctly. So that in the process of constructing the concept S1 experienced an assimilation process.

The thinking process of student 2 (S2) in constructing the concept of fractions which are represented by circles, rectangles, and number lines is also going well. In the process of forming the understanding, S2 can immediately understand which questions without repeating a lot in reading the questions. For the process of forming opinions, in this case S2 uses the scheme he has to solve the questions given. As for drawing conclusions, S2 uses the plan he has to solve questions confidently and correctly. So that in the process of constructing the concept S2 underwent an assimilation process.

The thinking process of student 3 (S3) in constructing the concept of fractions which are represented by circles, rectangles, and number lines is not going well. In the process of forming the understanding, S3 cannot immediately understand the questions given. S3 needs to understand the questions repeatedly. Then in the process of forming opinions S3 modifies the scheme it has to solve the given problem. As for drawing conclusions S3 uses the plan that has been made, but S3 is unable to realize the plan. As the result, in the process of constructing the concept S3 goes through an accommodation process.

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