WHAT IS A GEOMETRIC SHAPE? PRESCHOOL AND PRIMARY PRE-SERVICE TEACHER'S MISCONCEPTIONS

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Abstract

The paper describes the results of an inquiry conducted on preschool and primary pre-service teachers in Slovakia. Pre-service teachers were given a test with a task to choose the odd one out of four planar figures and explain their choice. The paper focuses on one item with four geometric shapes: parallelogram, isosceles trapezoid, convex octagon, and concave octagon. All these figures have a line of symmetry but a parallelogram. The study investigated whether pre-service teachers could discriminate one non-symmetrical figure from axially symmetrical figures. Surprisingly, the findings revealed unexpected misconceptions of pre-service teachers concerning the comprehension of the term "geometric shape".

Keywords: pre-service teachers, planar shapes, misconceptions

1. Introduction

Geometry plays a significant role in mathematics education. Based on the *Principles and Standards for School Mathematics*, to study geometry means to learn about geometric shapes and structures and how to analyse their characteristics and relationships (NCTM 2000, p. 41). Geometry is an appropriate tool for higher-order thinking skills development, too, since it calls for discovering similarities between geometric shapes, consciously using logic properties such as transitivity or asymmetry (based on deductive cognitive process), proving properties and statements by experimenting and formal logic, using analogies and reasoning in solving problems, etc. Geometry also holds an unchangeable position within mathematics education due to its unique nature of abstract constructs visualisation. Therefore, geometric constructs may be used in mathematics education as iconic representations (based on Bruner's theory of learning modes), e.g., a rectangle for multiplication; a rectangle or circle for fractions; a line for representing numbers, addition, and subtraction; points for representing various items in the combinatoric tasks, etc.

Shapes are a substantial part of teaching and learning geometry. Usually, geometric shapes (or figures) are classified as two-dimensional or three-dimensional. From Euclidean geometry's point of view, two-dimensional shapes lie on a plane, so they have length and height. In contrast, three-dimensional shapes (also called solid shapes) have one more spatial characteristic – a width. We will further focus on two-dimensional figures.

Two-dimensional figures are studied by plane geometry. Learning about those figures relates to learning about the properties of their sides, angles, diagonals, heights, areas, and so on. The content for learning and teaching plane geometry in Slovakia is specified in the *National Educational Program* (National Institute for Education, n.d.), covering preschool through high school educational content and goals all students should reach. With a focus on two-dimensional shapes, Slovak pupils are becoming familiar with them since the preschool stage. They learn to identify, draw, and compose a picture of discs, squares, rectangles, and triangles. Later in primary school, pupils become familiar with polygons (in general) and circles. They should know to describe the properties of all these figures; distinguish one from another; scale squares and rectangles up and down (in a square grid); calculate a perimeter of a triangle, square, and rectangle; know what points do and do not belong to a particular figure, and so on. When they proceed to the lower secondary stage, they learn more parallelograms such as rhombus, rhomboid, and trapezoid. Also, specific types of triangles are introduced (right, acute, obtuse, equilateral, and isosceles), and they formally learn about an area of rectangle, square, and right triangle. During the higher secondary stage, pupils broaden their knowledge of two-dimensional shapes by using a coordinate system, solving problems involving trigonometry, proving the properties of planar shapes, and so on.

In our belief, learning the mentioned content is beneficial for thinking development. It is a prerequisite for not only students proceeding to study STEM college majors but also for any individual solving technically oriented real-life problems. Unfortunately, as it happens in many subjects and content areas, not all students thrive in learning about two-dimensional figures.

2. Misconceptions in the learning of geometry

One of many reasons for students struggling in plane geometry may be anchored in what their mental images of plane figures are. Tall & Vinner (1981, p. 152) define the mental image as "the total cognitive structure that is associated with the concept, which includes all the mental pictures and associated properties and process". Suppose one's mental image of a concept does not ally with its scientific meaning. In that case, we say they have built a misconception (according to the constructivist learning philosophy, see works of Piaget, Vygotsky, Bruner, Posner, etc.). The term misconception is closely related to the term preconception. Term preconception is equivalent to what Piaget (1952) calls a schema – an existing unit of knowledge. Schemas are under development, so they are never complete. They are formed by information about some construct, whereas the information has been processed through abstraction and generalisation in the learning process. As schemas, preconceptions are constantly reconstructed by integrating new information (Bertrand, 1998). In the learning process, a pupil's preconception is the whole spectrum of knowledge they have about some construct before the construct is formally introduced by a teacher (Rumanová & Záhorská, 2019). Some of these preconceptions may be inadequate, not related to what is intended to learn, or missing important pieces of information. Thus, they are called misconceptions. Since no preconceptions are ideal, there is a thin line between what we call a preconception and misconception. From our perspective, misconceptions are the results of the intentional educational process. We call something a misconception if and only if students went through educational content and comprehended it, not as the instructor had hypothesised. Thus, the reason why a misconception has been formed may relate to an inadequate educational experience of the student (Abouelftouh & Alkramiti, 2020) that was created and delivered by an educator.

There were found many students' misconceptions related to planar geometry. Research body shows that students might have difficulties both with defining and classifying geometric planar shapes (see, for example, Vinner, 1991; de Villiers, 1994; Currie & Pegg, 1998; de Villiers, 1998; Monaghan, 2000; Zaslavsky & Shir, 2005; Atebe & Schäfer, 2008; Kembitzky, 2009; Žilková, 2013), not excluding elementary school pre-service teachers (see Jones, Mooney & Harries, 2002; Pickreign, 2007; Fujita & Jones, 2007, Cunningham & Roberts, 2010; Hnatová & Mokriš, 2021). They might also have difficulties identifying a model of planar shapes. For instance, they might not recognise a square when its model is rotated such that its diagonals are in vertical and horizontal positions (Gunčaga, Tkačik & Žilková, 2017). This

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misconception occurs not only among primary school pupils but can persist in some students even to the secondary school stage (Rafiah & Ekawati, 2017). In another example, some pupils might overlook an important aspect when identifying rectangles – their vertices – and claim a shape that looks like a rectangle with arches instead of vertexes as a rectangle (Gunčaga, Tkačik & Žilková, 2017). Some elementary pre-service teachers may have a similar misconception. Particularly, Mokriš & Scholtzová (2016) discovered a misconception of incorrectly identifying a squircle as a square. Pre-service elementary teachers also tend to make mistakes when asked to recognise an element of a geometric shape, such as the height of a triangle, a circular sector, and so on (Hnatová & Mokriš, 2021). We hypothesise that some credit for such misconceptions may be given to teachers for going through a myriad of math concepts rather formally than paying attention to a couple of central ideas.

One way of dealing with misconceptions early on is regular assessment of students' mental images (Ketterlin-Geller & Yovanoff, 2009; Tóthová, 2014). There are more ways of identifying or diagnosing individuals' mental models (preconceptions, misconceptions). The primary methods are a qualitative diagnosis that focuses on identifying misconceptions, and a quantitative diagnosis that allows studying their frequency within a group of students, comparing various groups of students, focusing on changes and further course of building concepts (Doulík & Škoda, 2003; Štrauch & Hanč, 2017). To obtain data, instruments such as questioning students, interviews, tests, multiple-choice questionnaires with incorporated distractors, concept maps, and writing prompts may be used (Drake & Amspaugh, 1994; Masingila & Prus-Wisniowska, 1996; Morrison, 1999; Ketterlin-Geller & Yovanoff, 2009; Štrauch & Hanč, 2017).

3. Methodology

3.1. Aim and data collection

This study initially aimed to investigate pre-service preschool and primary teachers' ability to discriminate figures that are not axially symmetrical from axially symmetrical figures. To do so, we developed four items test where each item consists of four planar figures. Pre-service teachers were asked to choose the odd one out in each item, whereas axial symmetry was not mentioned. Three of these figures were axially symmetrical; the fourth one was not. Moreover, students had to explain their choice. We hypothesised that most students would remember and embrace the concept of axial symmetry when analysing and selecting an odd figure. The test was distributed to students in two forms: a paper form and an electronic form. This paper focuses on one of four items (see Fig. 1) that consists of (a) a parallelogram, (b) an isosceles trapezoid, (c) a concave equilateral octagon, and (d) a convex octagon with pairs of equilateral sides. Figures (b), (c), and (d) are axially symmetrical; figure (a) is not.

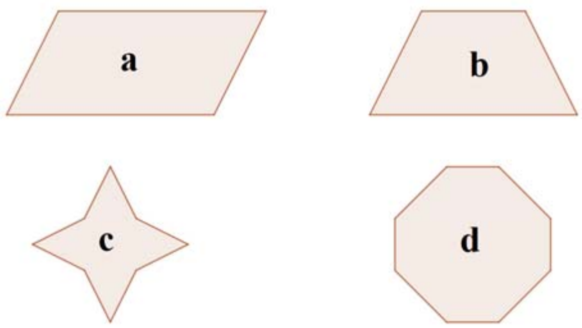

Figure 1. An item with three axially symmetrical plane figures (one is not axially symmetrical)

3.2. Participants

The group consisted of 66 preschool and primary school pre-service teachers in their first year of study in Slovakia. The group was consistent in all pre-service teachers studying at the same university. From the group, 22 participants filled out the paper test administrated by the author, and 44 completed the electronic test in their home environment. The electronic test was sent to 80 students, whereas 44 tests were filled and sent back. The paper test was administered in the classroom. The response rate of the electronic test was 55 %, and the response rate of the paper test was 100 %.

The participants were chosen based on cluster sampling, convenience sampling, and voluntary response sampling.

Participants had not gone through any geometry college courses then, although they took a Mathematical Literacy course in the previous semester in which they learned about some planar geometry concepts. None of the participants chose to pass the school-leaving exam from mathematics for upper secondary education. All 66 participants were girls, so there was no need to discuss potential differences in results based on gender. Due to missing answers, only 61 tests were examined (21 paper versions, 40 electronic versions) out of 66 submitted tests.

Participants' age inquiry was not conducted; participants' age average can be estimated as 19–20 years.

3.3. Data analysis

The received answers were transcribed into MS Excel, based on which eight meaning categories were created (see Chart 1). We used both quantitative and qualitative means to process and analyse the data.

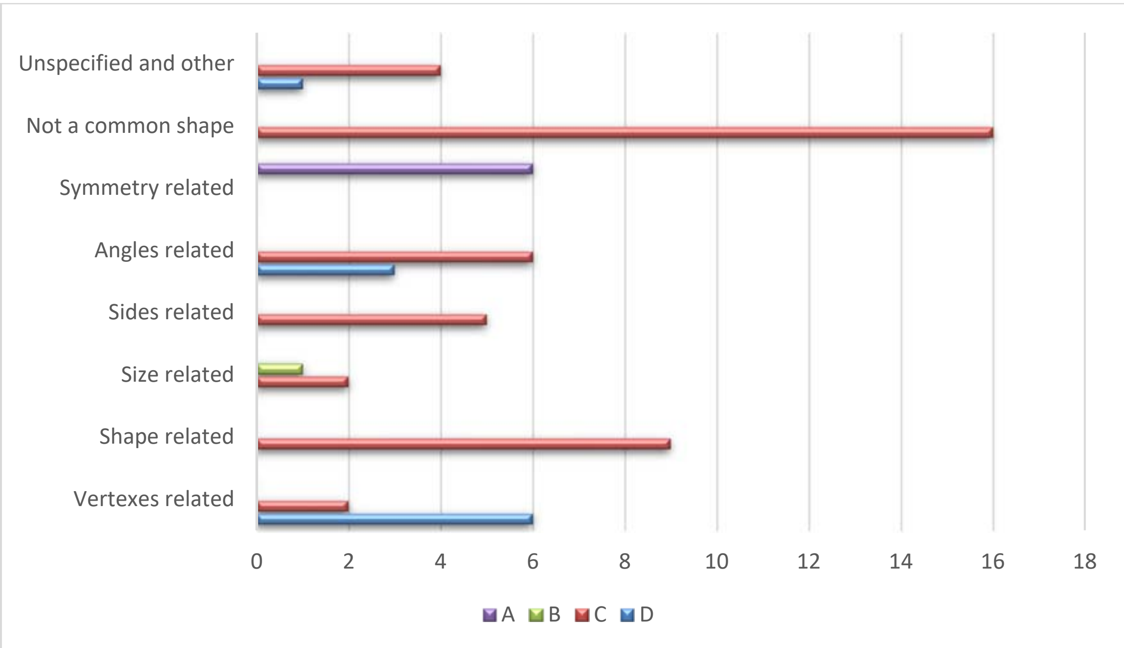

Chart 1. Visualisation of answers categorised into eight categories

We pursued a holistic approach in the data analysis process and focused on the most frequent responses. In the process of analysis, we further categorised answers from the category "not a common shape" into two subgroups: "not a common geometric shape" and "not a geometric shape" since it gives us interesting results for further discussion.

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3.4. Findings

The results show a prevalence of answers in favour of figure C (73.8 %), whereas the least dominant were the answers in favour of figure B (1.6 %). All pre-service teachers who chose figure A (9.8 %) explained their choice related to symmetry. The most numerous group were students who chose figure C (26.2 %), whereas their choice was supported by the explanation "not being a common shape". Furthermore, 16.4 % of all students explained choosing figure C as "it is not a geometric shape at all". Hence, this choice appears to be the most frequent in the collected data set; our initial hypothesis was not approved.

Another obtained result points to the relatively mediocre level of willingness of students to spend their time participating in research inquiry, as 45 % of asked students did not return the test.

4. Discussion

The study aimed to determine whether the property of axial symmetry embodied in pictorial models of specific geometric shapes can be recognised by preschool and primary pre-service teachers at the beginning of their professional preparation. Even though the results showed that some students did observe this property and used it as a discriminative factor, we expected a more significant number of students. Based on obtained answers, we see potential downsides of the presented item that could steer students' attention another way.

1. Figure C is the only one that is concave.
2. None of figure C's sides is horizontal.
3. Exposed figures do not have an equal number of sides.
4. Figure D is the only one with all angles obtuse.
5. Figure C is not commonly presented to students in geometry courses.

The mentioned list results from qualitative analysis and can be used to improve the testing instrument in further research. Not only data to improve the test were obtained. The research results discovered the misconception of some preschool and primary school pre-service teachers centred on the concept of a *geometric shape*. The results imply that some students may think of a *geometric shape* only as some of those common shapes, such as triangles, parallelograms, circles, polygons, and so on. Thus, they understand a *geometric shape* as a general category that includes specific geometric shapes learned in school, excluding all shapes they are unfamiliar with; or that are not prototypical. This issue may be anchored in how educators discuss geometric shapes with pupils through the pure systematisation of geometric shapes without pointing to what is and is not a geometric shape. To be clear, the classification of geometric shapes plays an essential role in learning about specific shapes (Elia & Gagatsis, 2003) and is beneficial for proving and inferencing properties from more general shapes, such as parallelograms, to more particular shapes, such as squares (Fujita & Jones, 2007).

We assume that discussing only typical geometric shapes in the classroom without mentioning some non-prototypical shapes does not follow the learning principle and the necessity of conceptual learning (NCTM, 2000, p. 20). Reasonably, school mathematics must be centred on central ideas and concepts that learners are likely to face in the future. Another argument might be related to time constraints, and thus that learning "non-curricular" content may prevent mathematics teachers from spending time discussing content specified in curriculum standards. Nevertheless, lesson structure depends on the educator, too; a few minor instructions or activities on deepening conceptual understanding of a *geometric shape* should not cause any harm either to learners or the run of the math course. Finally, we suggest that students should be acquainted at some level with what is and what is not a geometric shape.

This brings us to the heart of the problem: the definition of a *two-dimensional geometric shape*. Mathematics textbooks usually do not define what *geometric shape* means. Instead, this term is introduced through concrete types of geometric shapes such as lines, triangles, squares, rectangles, quadrilaterals, and so on. Scanning various math textbooks, we did not find any precise definition of geometric shape or non-model of a geometric shape (geometric figure). Euclid, in his work *Elements*, defines a figure as something "which is contained by some boundary or boundaries", whereas "a boundary is that which is the extremity of something" and "the extremities of a surface are lines" (Fitzpatrick, 2008, p. 6). A different transcription of Euclid's *Elements* says that "a figure is a surface enclosed on all sides by a line or lines" whereas "a line is length without breadth" (Byrne 1847, p. XVIII-XIX). Since those two transcriptions of Euclid's work do not specify whether these lines are straight or curved (despite he deals in his work with the term *straight line*), both types can be accepted. Thus, two-dimensional figures can be defined as an area closed by its boundary, whereas the boundary may consist of a combination of curves and straight lines. There is also a different way of defining a *figure* that uses a topological approach. Based on this approach, the interior points, boundary points, and exterior points of a figure can be defined as follows (Medek, Mišík & Šalát, 1975):

- point $X \in E_2$ is called an interior point of figure F if there exists a ball centred at the point X, that lies entirely in figure F,
- point $X \in E_2$ is called an exterior point of figure F if there exists a ball centred at the point X, that has an empty intersection with figure F,
- point $X \in E_2$ is called a boundary point of figure F if any ball centred at the point X, consists of both interiors and exterior points of figure F.

Either of these two definitions may be included in learning about geometric shapes, depending on the current level of students' cognitive development. Moreover, besides the importance of introducing the *planar shape* definition, it is essential to engage students in relevant activities. Thus, we suggest following instructions.

1. Draw a few arbitrary planar shapes that have 4 sides each. Compare your shapes with your classmate. What shapes do you have in common? What shapes are unique? How could we name such a shape?
2. There are three planar shapes in the picture. Write down all properties of those shapes you can think of. Put yourselves in groups and summarise your ideas.
3. There is a list of properties one planar shape should have. Use those properties to draw the corresponding shape. How many different shapes can you draw?

The mentioned instructions are quite general and not finite. They are supposed to serve rather as an inspiration than the limited list of possibilities for a math teacher.

5. Conclusion

Starting in preschool, shapes are discussed as common features of real objects that people can observe, touch, and model. They can be effortlessly compared to each other to determine how similar or different they are (Morgenstern et al., 2021). Young pupils should be confronted with shapes in activities where they must observe and describe various shapes. Based on that, pupils start noticing the properties of those shapes. Learning about geometric shapes, their properties, and general logic properties in learning geometry can occur through activities such as combining or cutting apart shapes for a new shape (NCTM, 2000, p. 98). If geometric content is taught inappropriately, students' mental models may differ from the scientific meaning in some essential attributes.

The study was conducted on novice preschool and primary pre-service teachers. Thus, their educational background differs from case to case. The survey results revealed unexpected students' misconceptions about the term *geometric shape*. Notably, some students may think of geometric shapes as just as common shapes they are typically learning about. We believe that the source of this misconception is an extreme accent on common shapes' hierarchisation (e. g., parallelograms, squares, triangles …) without discussing those shapes that do not fit into any of the shapes' categories.

The result of the study shows the need for inquiry about pre-service teachers' conceptions. Just as many other study results (e. g. Bulut & Bulut, 2012; Molitoris Miller, 2018), systematic inquiry serves as a basis for the revision of relevant mathematics courses within teacher preparation programs. We suggest that students should be confronted with more than just the "typical" hierarchy of two-dimensional geometric shapes.

References

Abouelftouh, D., & Alkramiti, M. (2020). Misconceptions of Mathematics and Its Relationship to The Learning Pleasure among Elementary School Students in Saudi Arabia. *Elementary Education Online*, 19 (4), 303–321. doi:10.17051/ilkonline.2020.04.131.

Atebe, H. U., & Schäfer, M. (2008). "As soon as the four sides are all equal, then the angles must be 90° each". Children's misconceptions in geometry. *African Journal Of Research In Mathematics, Science And Technology Education*, *12*(2), 47–65. https://doi.org/10.1080/10288457.2008.10740634.

Bertrand Y. (1998). Soudobé teorie vzdělávání. Praha: Portál.

Bulut, N., & Bulut, M. (2012). Development of Pre-Service Elementary Mathematics Teachers' Geometric Thinking Levels Through an Undergraduate Geometry Course. *Procedia - Social And Behavioral Sciences*, *46*, 760–763. https://doi.org/10.1016/j.sbspro.2012.05.194.

Byrne, O. (1847). The First Six Books of the Elements of Euclid with Coloured Diagrams and Symbols. London: William Pickering. https://www.math.uci.edu/~ndonalds/Elements-I-VI.pdf.

Cunningham, R. F., & Roberts, A. (2010). Reducing the mismatch of geometry concept definitions and concept images held by pre-service teachers. *IUMPST: The Journal. Vol 1 (Content Knowledge)*. http://www.k-12prep.math.ttu.edu/journal/1.contentknowledge/cunningham01/article.pdf.

Currie, P., & Pegg, J. (1998). Investigating students understanding of the relationships among quadrilaterals. In: C. Kanes, M. Goos, & E. Warren (Eds.), *Teaching Mathematics in New Times, Proceedings of the 21th Annual Conference of the Mathematics Education Research Group of Australasia Incorporated, Gold Coast, Australia 5 – 8 July*, Volume 1, (p. 177–184). https://www.merga.net.au/Public/Publications/Annual_Conference_Proceedings/1998_MERGA_CP.aspx.

de Villiers, M. (1994). The role and function of a hierarchical classification of quadrilaterals. *For the Learning of Mathematics*, 14(1), 11−18. https://flm-journal.org/Articles/58360C6934555B2AC78983AE5FE21.pdf.

de Villiers, M. (1998). To teach definitions in geometry or teach to define? In: A. Olivier, & K. Newstead (Eds.), *Proceedings of the 22nd Conference of the International Group for the Psychology of Mathematics Education*, 2, (pp. 248−255). http://academic.sun.ac.za/mathed/174/MichaelDefining.pdf.

Doulík, P., & Škoda, J. (2003). Tvorba a ověření nástrojů kvantitativní diagnostiky prekonceptu a možnosti jejího vyhodnocení. *Pedagogika*, 53(2), 177−189. https://pages.pedf.cuni.cz/pedagogika/?attachment_id=1917&edmc=1917.

Drake, B. M., & Amspaugh, L. B. (1994). What Writing Reveals in Mathematics. *Focus on Learning Problems in Mathematics*, 16(3), 43−50.

Elia, I., & Gagatsis, A. (2003). Young children's understanding of geometric shapes: The role of geometric models: The role of geometric models. *European Early Childhood Education Research Journal*, *11*(2), 43−61. https://doi.org/10.1080/13502930385209161.

Fitzpatrick, R. (2008). *Euclid's Elements of Geometry - The Greek text of J. L. Heiberg (1883−1885)*. https://farside.ph.utexas.edu/books/Euclid/Elements.pdf.

Fujita, T., & Jones, K. (2007). Learners' understanding of the definitions and hierarchical classification of quadrilaterals: towards a theoretical framing. *Research in Mathematics Education*, 9(1&2), 3−20. https://doi.org/10.1080/14794800008520167.

Gunčaga, J., Tkačik, Š., & Žilková, K. (2017). Understanding of Selected Geometric Concepts by Pupils of Pre-Primary and Primary Level Education. *European Journal of Contemporary Education*, 6(3), 497−515. https://doi.org/10.13187/ejced.2017.3.497.

Hnatová, J., & Mokriš, M. (2021). *Kalibrácia metakognitívneho monitorovania v profesijnej matematickej príprave budúcich učiteľov: Presnosť a skreslenie hodnotenia*. Prešov: Vydavateľstvo Prešovskej univerzity.

Jones, K., Mooney, C., & Harries, T. (2002). Trainee primary teachers' knowledge of geometry for teaching. *Proceedings of the British Society for Research into Learning Mathematics*, 22(2), 95−100.

Kembitzky, K. A. (2009). *Addressing Misconceptions in Geometry through Written Error Analyses*. [Doctoral dissertation, The Ohio State University]. https://etd.ohiolink.edu/apexprod/rws_etd/send_file/send?accession=osu1259169709&disposition=inline.

Ketterlin-Geller, L. R., & Yovanoff, P. (2009). Diagnostic assessments in mathematics to support instructional decision making. *Practical Assessment, Research and Evaluation*, 14(16), https://doi.org/10.7275/vxrk-3190.

Masingila, J., & Prus-Wisniowska, E. (1996). Developing and assessing mathematical understanding in calculus through writing. In: P. Elliot, & M. Kenney (Eds.), *National Council of Teachers of Mathematics 1996 yearbook: Communication in mathematics, K-12 and beyond* (pp. 95−104): Reston, VA: National Council of Teachers of Mathematics.

Medek, V., Mišík, L., & Šalát, T. (1983). *Repetitórium stredoškolskej matematiky, 3. vydanie*. Bratislava: Alfa.

Mokriš, M., & Scholtzová, I. (2016). Miskoncepcie v identifikácii rovinných geometrických útvarov u budúcich učiteľov-elementaristov. *Studia Scientifica Facultatis Paedagogicae: Universitatis Catholicae in Ružomberok*. Ružomberok, (pp. 141−146).

Molitoris Miller, S. (2018). An analysis of the form and content of quadrilateral definitions composed by novice pre-service teachers. *The Journal Of Mathematical Behavior*, *50*, 142−154. https://doi.org/10.1016/j.jmathb.2018.02.006.

Monaghan, F. (2000). What difference does it make? Children views of the difference between some quadrilaterals. *Educational Studies in Mathematics*, 42(2), 179–196. https://doi.org/10.1023/A:1004175020394.

Morgenstern Y, Hartmann F, Schmidt F, Tiedemann H, Prokott E, Maiello G, & Fleming, R. W. (2021). An image-computable model of human visual shape similarity. *PLoS Comput Biol* 17(6): e1008981. https://doi.org/10.1371/journal.pcbi.1008981.

Morrison, J. A. (1999). *Investigating Teachers' Understanding and Diagnosis of Students' Preconceptions in the Secondary Science Classroom*. [Doctoral thesis, Oregon State University]. https://ir.library.oregonstate.edu/downloads/vq27zr61k.

National Council of Teachers of Mathematics (NCTM). (2000). *Principles and Standards for School Mathematics*. https://www.nctm.org/Standards-and-Positions/Principles-and-Standards/.

National Institute for Education. (n. d.). *National Educational Program.* Retrieved June 20, 2022. https://www.statpedu.sk/sk/svp/inovovany-statny-vzdelavaci-program/.

Piaget, J., & Cook, M. T. (1952). *The origins of intelligence in children*. New York, NY: International University Press.

Pickreign, J. (2007). Rectangle and rhombi: how well do pre-service teachers know them? *Issues in the Undergraduate Mathematics Preparation of School Teachers*, Volume 1: Content Knowledge. http://www.k-12prep.math.ttu.edu/journal/1.contentknowledge/pickreign01/article.pdf.

Rafiah, H., & Ekawati, A. (2017). Misconceptions of the Students with High Mathematical Creative Thinking Level in Solving the Geometric Shapes Problems. In: *Proceedings of the 5th SEA-DR (South East Asia Development Research) International Conference 2017 (SEADRIC 2017)*, (pp. 155−158). https://doi.org/10.2991/seadric-17.2017.33.

Rumanová, L., & Záhorská, J. (2019). Chyby v riešeniach vybraných úloh z geometrie. *Acta Mathematica Nitriensia*, 5(1), 23-29. http://www.amn.fpv.ukf.sk/papers/amn_5_2/3_Rumanova_AMN_vol_5_no_2_2019.pdf.

Štrauch, P., & Hanč, J. (2017). Kvantitatívna diagnostika miskoncepcií v prírodovednom vzdelávaní. *Edukácia*, 2(1), 291−302. https://www.upjs.sk/public/media/15903/Strauch_Hanc.pdf.

Tall, D. O., & Vinner, S. (1981). Concept image and concept definition in mathematics, with special reference to limits and continuity. *Educational Studies in Mathematics*, 12(2), 151−169.

Tóthová, R. (2014). *Konštruktivistický prístup vo výučbe ako možnosť rozvoja myslenia žiakov*. Bratislava: Metodicko-pedagogické centrum. https://mpc-edu.sk/sites/default/files/projekty/vystup/tothova.pdf.

Vinner, S. (1991). The role of definitions in the teaching and learning of mathematics, in: D. O. Tall (Ed.), *Advanced Mathematical Thinking*. Dordrecht: Kluwer Academic Publishers.

Zaslavsky, O., & Shir, K. (2005). Students' conceptions of a mathematical definition. *Journal for Research in Mathematics Education*, 36(4), 317−346.

Žilková, K. (2013). *Teória a prax geometrických manipulácií v primárnom vzdelávaní*. Praha: Powerprint.