DIVISIBILITY AND MS EXCEL

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Abstract

Divisibility plays a crucial role in arithmetic, number theory and modular arithmetic. The orientation in the divisibility is needed to solve many tasks from everyday life. Thus, divisibility is an important part of mathematics curriculum. Children are beginning to get acquainted with the basic concepts and principles of divisibility since the age of 8. It seems that the teaching of divisibility can be interesting when using MS Excel. In this way, we can also increase pupils' digital literacy. In our paper, we will focus on possibilities of using the MOD function in MS Excel. We will solve some word problems.

Keywords: divisibility, modular arithmetic, MS Excel, word problems.

1. Introduction

Divisibility is an important part of mathematics curriculum. Pupils are beginning to get acquainted with the basic concepts and principles of divisibility since the age of 8. Children of elementary school know that a (concrete) natural number a divides a (concrete) natural number b, if there is a (concrete) natural number c such that $b = a \cdot c$. They also know how to find, for two (concrete) natural numbers a and b, the quotient q and the remainder r such that $b = a \cdot q + r$. Pupils become acquainted with concepts such as the greatest common divisor or the least common multiple at the age of 12, but at the age 8-11 they prepare for it.

Using MS Excel to solve some examples can make the divisibility learning more attractive. In MS Excel, concerning divisibility, we can use the following functions ("Excel function guide").

- MOD function: returns the remainder after division (returns the remainder after the first number is divided by a divisor). For example MOD(18;5) returns 3 or MOD(2020;37) returns 22.

- GCD function: returns the greatest common divisor of two or more natural numbers. We recall that the greatest common divisor is the largest positive integer which divides all supposed natural numbers. For example, MOD(16;28) returns 4 or MOD(7;33) returns 1 or MOD(72;360;108) returns 36.

- LCM function: returns the least common multiple of two or more natural numbers. We recall that the least common multiple is the smallest positive integer which is a multiple of all supposed natural numbers. For example, LCM(12;8) returns 24 or LCM(7;13) returns 91 or LCM(1348;1415;2020) returns 192 649 420.

For teaching divisibility of pupils aged 8-11 we can use the MOD function above all. In our paper, we will show some examples of word problems that can be solved by means of the MOD function. We will use also the next functions in MS Excel ("Excel function guide")..

- IF function: returns one value if the condition is true, or another value if the condition is false. For example, IF(MOD(72;8)=0;1;0) returns 1 because the number 8 divides the number 72. On the other hand IF(MOD(22;3)=0;1;0) returns 0 because the number 3 does not divide the number 22.

- INT function: returns the integer part of a decimal number by rounding down to the integer. For example, INT(2,85) returns 2 or INT(29/7) returns 4.

2. Word problems

In this section, we will show how we can solve some word problems concerning divisibility using MS Excel. The given examples were solved in ("Divisibility – math word problems") using procedures with which pupils aged 8-11 are usually unfamiliar (such as finding the least common multiple using prime factorization).

Example 1 ("Divisibility – math word problems", Example 4).

Paľo, Jano, Karol, and Rišo were doing an intelligence test. Paľo correctly answered half of the questions plus 7 questions, Jano to one third plus 18 questions, Karol to one quarter plus 21 questions and Rišo to one fifth plus 25 questions.

After the test, Karol said: "I feel like I did quite well. I don't know how many questions were in the test, but the number of them was less than or equal to 100. How many questions were in the test?

What was the name of the boy who had the most correct answers? How many correct answers did he have?

Solution. It follows from the text of word problem that the number of questions is greater than 25 and less than or equal to 100. We also know that the number of questions is divisible by 2, 3, 4 and 5. Now, we can use MS Excel as it is shown in Tables 1 and 2.

Table 1. Example 1, used functions in MS Excel.

	A	B	C	D	E	F
1	26	=MOD(A1;2)	=MOD(A1;3)	=MOD(A1;4)	=MOD(A1;5)	=IF(B1+C1+D1+E1=0;"YES";"NO")
2	27	=MOD(A2;2)	=MOD(A2;3)	=MOD(A2;4)	=MOD(A2;5)	=IF(B2+C2+D2+E2=0;"YES";"NO")
3	28	=MOD(A3;2)	=MOD(A3;3)	=MOD(A3;4)	=MOD(A3;5)	=IF(B3+C3+D3+E3=0;"YES";"NO")
4	29	=MOD(A4;2)	=MOD(A4;3)	=MOD(A4;4)	=MOD(A4;5)	=IF(B4+C4+D4+E4=0;"YES";"NO")
5	30	=MOD(A5;2)	=MOD(A5;3)	=MOD(A5;4)	=MOD(A5;5)	=IF(B5+C5+D5+E5=0;"YES";"NO")
6	31	=MOD(A6;2)	=MOD(A6;3)	=MOD(A6;4)	=MOD(A6;5)	=IF(B6+C6+D6+E6=0;"YES";"NO")
7	32	=MOD(A7;2)	=MOD(A7;3)	=MOD(A7;4)	=MOD(A7;5)	=IF(B7+C7+D7+E7=0;"YES";"NO")
8	33	=MOD(A8;2)	=MOD(A8;3)	=MOD(A8;4)	=MOD(A8;5)	=IF(B8+C8+D8+E8=0;"YES";"NO")
.	.	.	.	.	.	.
32	57	=MOD(A32;2)	=MOD(A32;3)	=MOD(A32;4)	=MOD(A32;5)	=IF(B32+C32+D32+E32=0;"YES";"NO")
33	58	=MOD(A33;2)	=MOD(A33;3)	=MOD(A33;4)	=MOD(A33;5)	=IF(B33+C33+D33+E33=0;"YES";"NO")
34	59	=MOD(A34;2)	=MOD(A34;3)	=MOD(A34;4)	=MOD(A34;5)	=IF(B34+C34+D34+E34=0;"YES";"NO")
35	60	=MOD(A35;2)	=MOD(A35;3)	=MOD(A35;4)	=MOD(A35;5)	=IF(B35+C35+D35+E35=0;"YES";"NO")
36	61	=MOD(A36;2)	=MOD(A36;3)	=MOD(A36;4)	=MOD(A36;5)	=IF(B36+C36+D36+E36=0;"YES";"NO")
37	62	=MOD(A37;2)	=MOD(A37;3)	=MOD(A37;4)	=MOD(A37;5)	=IF(B37+C37+D37+E37=0;"YES";"NO")
38	63	=MOD(A38;2)	=MOD(A38;3)	=MOD(A38;4)	=MOD(A38;5)	=IF(B38+C38+D38+E38=0;"YES";"NO")
.	.	.	.	.	.	.
73	98	=MOD(A73;2)	=MOD(A73;3)	=MOD(A73;4)	=MOD(A73;5)	=IF(B73+C73+D73+E73=0;"YES";"NO")
74	99	=MOD(A74;2)	=MOD(A74;3)	=MOD(A74;4)	=MOD(A74;5)	=IF(B74+C74+D74+E74=0;"YES";"NO")
75	100	=MOD(A75;2)	=MOD(A75;3)	=MOD(A75;4)	=MOD(A75;5)	=IF(B75+C75+D75+E75=0;"YES";"NO")

Table 2. Example 1, results in MS Excel.

	A	B	C	D	E	F
1	26	0	2	2	1	NO
2	27	1	0	3	2	NO
3	28	0	1	0	3	NO
4	29	1	2	1	4	NO
5	30	0	0	2	0	NO
6	31	1	1	3	1	NO
7	32	0	2	0	2	NO
8	33	1	0	1	3	NO
.	.	.	.	.	.	.
32	57	1	0	1	2	NO
33	58	0	1	2	3	NO
34	59	1	2	3	4	NO
35	60	0	0	0	0	YES
36	61	1	1	1	1	NO
37	62	0	2	2	2	NO
38	63	1	0	3	3	NO
.	.	.	.	.	.	.
73	98	0	2	2	3	NO
74	99	1	0	3	4	NO
75	100	0	1	0	0	NO

Table 2 gives that there were 60 questions in the test. We can easily calculate that Paľo answered correctly $\frac{1}{2} \cdot 60 + 7 = 37$ questiones, Jano answered correctly $\frac{1}{3} \cdot 60 + 18 = 38$ questions, Karol answered correctly $\frac{1}{4} \cdot 60 + 21 = 36$ and Rišo answered correctly $\frac{1}{5} \cdot 60 + 25 = 37$. Thus, the name of the boy who had the most correct answers is Jano.

Pupils may notice that if 4 divides a number, then 2 divides this number too. Thus, the column B in Tables 1 and 2 can be omitted.

Example 2. ("Divisibility – math word problems", Example 4).
How many are two-digit natural numbers that have the sum of the digits 9?

Solution. We can use Excel functions INT, MOD and IF, see Tables 3 and 4.

Table 3. Example 2, used functions in MS Excel.

	A	B	C	D	E
1	10	=INT(A1/10)	=MOD(A1;10)	=B1+C1	IF(D1=9; "YES";"NO")
2	11	=INT(A2/10)	=MOD(A2;10)	=B2+C2	IF(D2=9; "YES";"NO")
3	12	=INT(A3/10)	=MOD(A3;10)	=B3+C3	IF(D3=9; "YES";"NO")
4	13	=INT(A4/10)	=MOD(A4;10)	=B4+C4	IF(D4=9; "YES";"NO")
5	14	=INT(A5/10)	=MOD(A5;10)	=B5+C5	IF(D5=9; "YES";"NO")
6	15	=INT(A6/10)	=MOD(A6;10)	=B6+C6	IF(D6=9; "YES";"NO")
7	16	=INT(A7/10)	=MOD(A7;10)	=B7+C7	IF(D7=9; "YES";"NO")
8	17	=INT(A8/10)	=MOD(A8;10)	=B8+C8	IF(D8=9; "YES";"NO")
9	18	=INT(A9/10)	=MOD(A9;10)	=B9+C9	IF(D9=9; "YES";"NO")
.	.	.	.	.	.
87	96	=INT(A87/10)	=MOD(A87;10)	=B87+C87	IF(D87=9; "YES";"NO")
88	97	=INT(A88/10)	=MOD(A88;10)	=B88+C88	IF(D88=9; "YES";"NO")
89	98	=INT(A89/10)	=MOD(A89;10)	=B89+C89	IF(D89=9; "YES";"NO")
90	99	=INT(A90/10)	=MOD(A90;10)	=B90+C90	IF(D90=9; "YES";"NO")

Table 4. Example 2, results in MS Excel.

	A	B	C	D	E
1	10	1	0	1	NO
2	11	1	1	2	NO
3	12	1	2	3	NO
4	13	1	3	4	NO
5	14	1	4	5	NO
6	15	1	5	6	NO
7	16	1	6	7	NO
8	17	1	7	8	NO
9	18	1	8	9	YES
10	19	1	9	10	NO
.	.	.	.	.	.
18	27	2	7	9	YES
19	28	2	8	10	NO
.	.	.	.	.	.
27	36	3	6	9	YES
28	37	3	7	10	NO
.	.	.	.	.	.
36	45	4	5	9	YES
37	46	4	6	10	NO
.	.	.	.	.	.
45	54	5	4	9	YES
46	55	5	5	10	NO
.	.	.	.	.	.
54	63	6	3	9	YES
55	64	6	4	10	NO
.	.	.	.	.	.
63	72	7	2	9	YES
64	73	7	3	10	NO
.	.	.	.	.	.
72	81	8	1	9	YES
73	82	8	2	10	NO
.	.	.	.	.	.
81	90	9	0	9	YES
82	91	9	1	10	NO
.	.	.	.	.	.
90	99	9	9	18	NO

Using Table 4, we can answer that there are 9 two-digit natural numbers that have the sum of the digits 9: 18, 27, 36, 45, 54, 63, 72, 81 and 90. Pupils may notice that the column D can be omitted if we use the formula =IF(B1+C1=9;"YES";"NO") in the last column.

Example 3. ("Divisibility – math word problems", Example 16).
The dance group formed groups of 4, 5, and 6 members. Always one dancer remained. How many dancers could there in the whole group?

Solution. The number of dancers is greater than or equal to 7. We are looking for the smallest number for which the remainder after dividing 4, 5 and 6 equals to 1. We will use Excel functions MOD and IF, see Tables 5 and 6. In Table 6 we will end as soon as YES appears in column E. Thus, the minimal number of dancers in the whole group is 61.

Pupils may notice, for example, that formula =IF(B1*C1*D1=3; "YES";"NO") in the column E cannot be replaced by the formula =IF(B1+C1+D1=3;"YES";"NO").

Table 5. Example 3, used functions in MS Excel.

	A	B	C	D	E
1	7	=MOD(A1;4)	=MOD(A1;5)	=MOD(A1;6)	IF(B1*C1*D1=1; "YES";"NO")
2	8	=MOD(A2;4)	=MOD(A2;5)	=MOD(A2;6)	IF(B2*C2*D2=1; "YES";"NO")
3	9	=MOD(A3;4)	=MOD(A3;5)	=MOD(A3;6)	IF(B3*C3*D3=1; "YES";"NO")
4	10	=MOD(A4;4)	=MOD(A4;5)	=MOD(A4;6)	IF(B4*C4*D4=1; "YES";"NO")
5	11	=MOD(A5;4)	=MOD(A5;5)	=MOD(A5;6)	IF(B5*C5*D5=1; "YES";"NO")
6	12	=MOD(A6;4)	=MOD(A6;5)	=MOD(A6;6)	IF(B6*C6*D6=1; "YES";"NO")
7	13	=MOD(A7;4)	=MOD(A7;5)	=MOD(A7;6)	IF(B7*C7*D7=1; "YES";"NO")
8	14	=MOD(A8;4)	=MOD(A8;5)	=MOD(A8;6)	IF(B8*C8*D8=1; "YES";"NO")
9	15	=MOD(A9;4)	=MOD(A9;5)	=MOD(A9;6)	IF(B9*C9*D9=1; "YES";"NO")
.	.	.	.	.	.

Table 6. Example 3, results in MS Excel.

	A	B	C	D	E
1	7	3	2	1	NO
2	8	0	3	2	NO
3	9	1	4	3	NO
4	10	2	0	4	NO
5	11	3	1	5	NO
6	12	0	2	0	NO
7	13	1	3	1	NO
8	14	2	4	2	NO
9	15	3	0	3	NO
.	.	.	.	.	.
52	58	2	3	4	NO
53	59	3	4	5	NO
54	60	0	0	0	NO
55	61	1	1	1	YES

3. Conclusion

We were interested in some word problems concerning divisibility that can be solved by pupils aged 10 to 11 by means of MS Excel. This may prepare pupils well to study divisibility in the next years.

Divisibility plays a crucial role in modular arithmetic. Recall ("Modular arithmetic", Wikipedia) that modular arithmetic is a system of arithmetic for integers, where numbers "wrap around" when reaching a certain value, called the modulus. In theoretical mathematics, modular arithmetic is extensively used in number theory, group theory, ring theory and abstract algebra. In applied mathematics, modular arithmetic is used, for example, in computer science and cryptography.

Modular arithmetic can be characterized by the following way ("The Irish Time", 2017): you may not know it but you use it every day.

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