ISSN 2694-8133

KANGAROO ON THE CHESSBOARD AS A DIDACTIC TOOL

Karel PASTOR
Palacký University Olomouc, Faculty of Education (Czech Republic)
karel.pastor@upol.cz

Abstract

The level of mathematical literacy can be significantly increased by means of board games as for example chess. Solving mathematical chess problems can develop combinatorial skills of pupils aged 6-11. Mathematical chess problems use chessboard or chess pieces. We will focus on a special piece named kangaroo that was introduced in the competition Mathematical Kangaroo. We will deal, among the others, with the domination problem of kangaroo, the independence problem of kangaroo and the kangaroo tour problem. These problems have been already solved for 4×4 and 6×6 chessboards in the previous papers, so we will be interested in 5×5 chessboard. The reduced chessboard is used because a smaller chessboard seems to be more accessible to pupils aged 6 to 11.

Keywords: Mathematical chess problems, combinatorics, Mathematical Kangaroo

1. Introduction

We have already studied mathematical chess problems in our previous papers (Pastor, 2019), (Pastor, 2020a), (Pastor, 2020b).

Recall that a mathematical chess problem is a mathematical problem that is formulated using a chessboard and chess pieces ("Mathematical chess problems", Wikipedia). For more details, see (Gik, 2019), (Chybová, 2017), (Watkins, 2004).

In our paper, we will focus on a special piece named kangaroo that was introduced in the competition Mathematical Kangaroo ("Matematický klokan 2015"). We will deal, among the others, with the domination problem of kangaroo, the independence problem of kangaroo and the kangaroo tour problem. We will use a reduced chessboard 5×5 because this chessboard seems to be more appropriate for pupils aged 6 to 11. Moreover, we will illustrate on some examples that also chess diagrams can develop combinatorial skills.

A kangaroo moves one square horizontally then three squares vertically or three squares horizontally then one square vertically, see Figure 1.

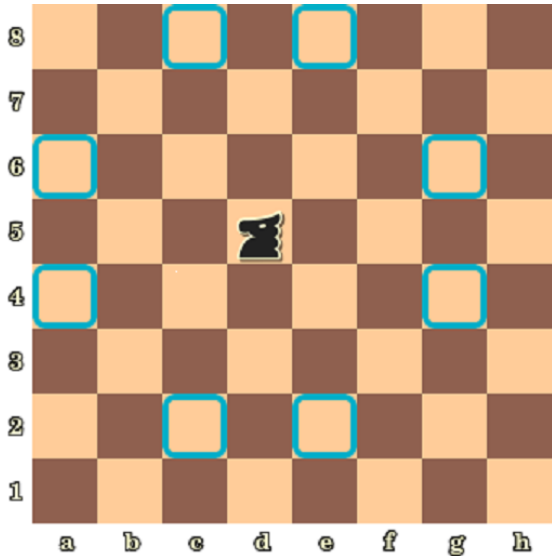

Figure 1. Moves of a kangaroo

2. Independence problem of kangaroo

Find the maximum number of kangaroos which can be placed on a chessboard so that none of the pieces attacks each other.

This problem has been already solved for the chessboard 6×6 (Pastor, 2019). It was shown that the maximum number is 18.

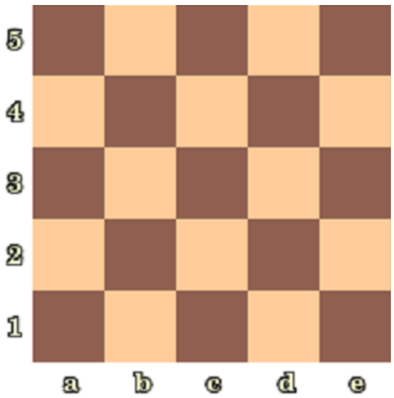

Figure 2. Chessboard 5×5

Now, let us consider the chessboard 5×5 (Figure 2). We can divide the board into two 2×5 rectangles, e.g. (a1, a2, a3, a4, a5, b1, b2, b3, b4, b5) + (c1, c2, c3, c4, c5, d1, d2, d3, d4, d5), and one 1×5 rectangle (e1, e2, e3, e4, e5). Since a kangaroo placed anywhere within one of these 2×5 rectangles can move exactly on one square of the considered rectangle, at most 5 kangaroos can be placed in each rectangle 2×5. Since there are two rectangles 2×5, at most 10 kangaroos can be placed on the considered two rectangles together. Considering the remaining 1×5 rectangle, we obtain that at most 15 kangaroos can be placed on the chessboard 5×5.

Figure 3 shows that it is actually possible to place 15 independent kangaroos on the chessboard 5×5.

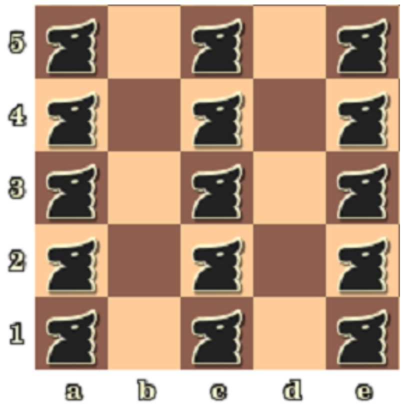

Figure 3. 15 kangaroos

3. Domination problem of kangaroo

Find a minimum number of kangaroos and place them on a chessboard in such a way, that all free squares of the board are attacked by at least one kangaroo.

This problem has been already solved for the chessboard 6×6 (Pastor, 2020a). It was shown that the minimum number is 8. Thus, for the chessboard 5×5, the minimum number is 8 or less, see Figure 4.

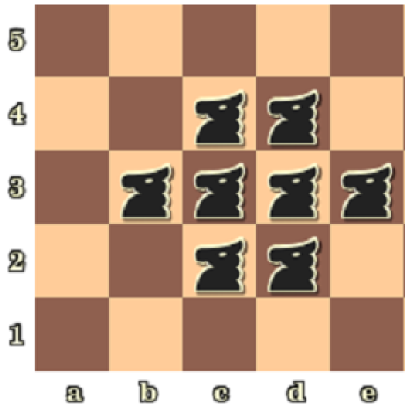

Figure 4. 8 kangaroos

It is an open question whether it is possible to place on the 5×5 chessboard less than 8 kangaroos in such a way, that all free squares of the board are attacked by at least one kangaroo.

4. Kangaroo tour problem

Find a tour for the kangaroo on the chessboard to visit all squares of the chessboard just once.

Because the kangaroo is only able to reach squares of one colour, the problem is reduced in terms of whether the kangaroo can visit all 13 black squares or all white 12 squares on the 5×5 chessboard.

Starting with the black variant, it is necessary to visit all corners of the 5×5 chessboard but, in this way, not all black squares could be visited, see e.g., Figure 5.

In contrast, the kangaroo can visit all the white squares on the 5×5 chessboard as shown in Figure 6.

2021, Vol. 3, No. 2
ISSN 2694-8133

7				3
	4		8	
				5
	6		2	
1				9

Figure 5. Black squares

	5		3	
8		10		12
	1		7	
6		4		2
	9		11	

Figure 6. White squares

5. Chess diagrams with kangaroo

Finishing my lecture, I would like to show some examples of chess diagrams with kangaroo.

Task 1. On the Figure 7, black to move and mate in one move. Notice that there are two kangaroos on the squares a5, e4, a white king on the square a8 and a black king on the square b6. You can use e.g. ("Rules of chess", Wikipedia) to recall the movements of chess pieces.

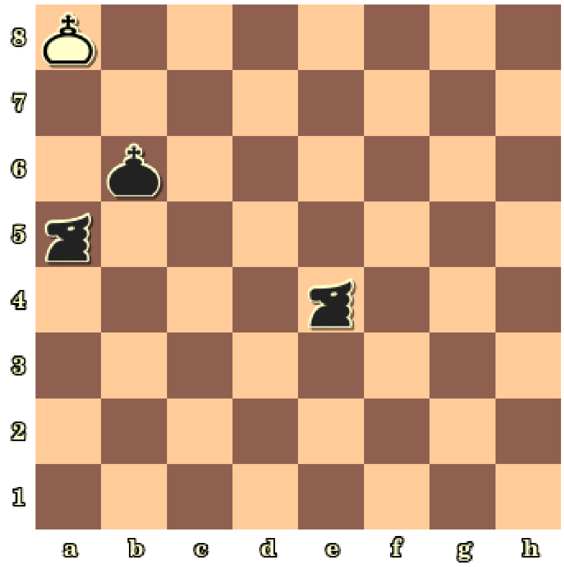

Figure 7. Task 1

Solution. See Figure 8.

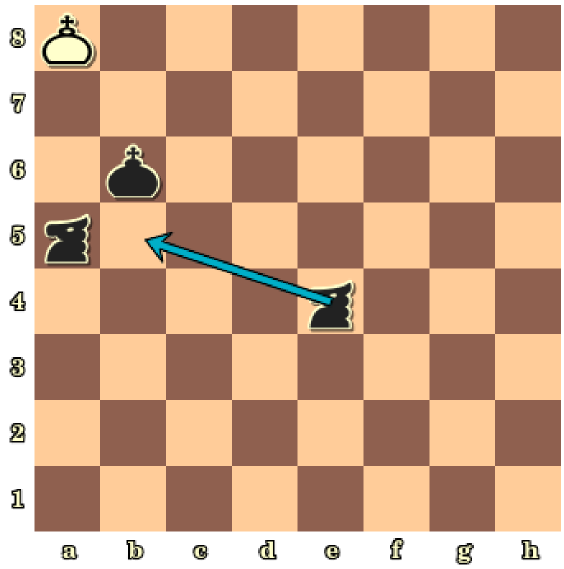

Figure 8. Solution of Task 1

Task 2. On the Figure 9, black to move and mate in one move. Notice that there are five white pawns on the squares a2, b2, c2, f2 and g3, two white rooks on the squares d1 and h1, a white queen on the square e7, a white king on the square b1, five black pawns on the squares a7, b7, c7, f5, g4, a black kangaroo on the square c5, a black queen on the square b6 and a black king on the square b8.

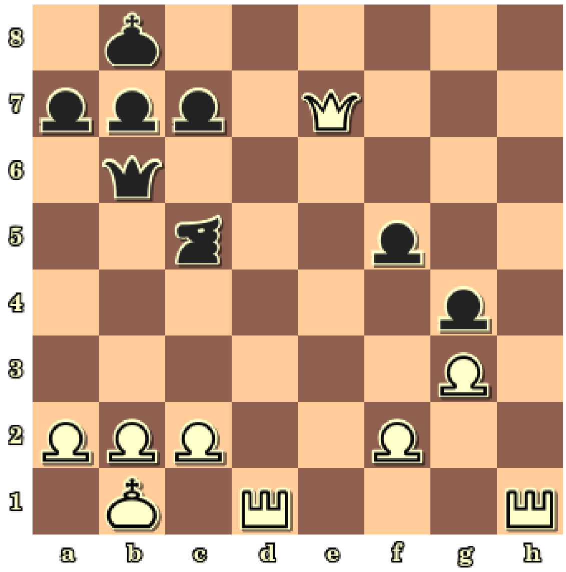

Figure 9. Task 2

Solution. See Figure 10.

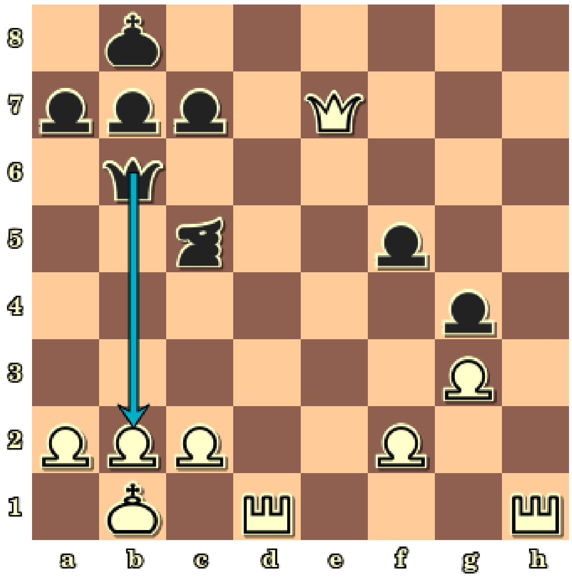

Figure 10. Solution of Task 2

6. Conclusion

We were interested in some chess mathematical problems with a special piece named kangaroo. We mainly used the 5×5 chessboard to make problems more accessible to children aged 6 to 11. To solve the previous problems pupils can use, for example, some computer application, see e.g. ("Chess Diagram Setup"). In our paper we have stated an open domination problem.

References

Chybová, L. (2017). *Šachové úlohy v kombinatorice.* [Diplomová práce]. Univerzita Karlova. http://kdm.karlin.mff.cuni.cz/diplomky/lucie_chybova_dp/sachove-ulohy.pdf.

Gik, E. (2019). *Šachmaty i matematika*. Retrieved 18-05-2021, http://golovolomka.hobby.ru/books/gik/index.shtml.

Pastor, K. (2019). Chess independence problems. *Elementary Mathematics Education Journal 1 (2).* Retrieved 18-05-2021, http://emejournal.upol.cz/Issues/Vol1No2/Pastor_2019_Vol1No2.pdf.

Pastor, K. (2020a). Chess domination problems. *Elementary Mathematics Education Journal 2 (1).* Retrieved 18-05-2021, http://emejournal.upol.cz/Issues/Vol2No1/Pastor_2020_Vol2No1.pdf.

Pastor, K. (2020b). Chess piece tour problems. *Elementary Mathematics Education Journal 2 (2).* Retrieved 18-05-2021, http://emejournal.upol.cz/Issues/Vol2No2/Vol2No2_Pastor2.pdf.

Watkins, J. (2004). *Across the Board: The Mathematics of Chessboard Problems*. Princeton University Press.

(n.d.). *Mathematical chess problems*. (Wikipedia). Retrieved 26-06-2019, https://en.wikipedia.org/wiki/Mathematical_chess_problem.

(n.d.). *Rules of chess*. (Wikipedia). Retrieved 26-06-2019, https://en.wikipedia.org/wiki/Rules_of_chess

(n.d). *Matematický klokan 2015*. Retrieved 26-06-2019, http://www.matematickyklokan.net/phocadownload/sborniky/sbornik_klokan_2015.pdf.

(n.d.). *Chess Diagram Setup*. Retrieved 26-06-2019, https://www.jinchess.com/chessboard/composer/.