DEVELOPING LOGICAL THINKING THROUGH TASKS IN MATHEMATICS TEXTBOOKS

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Abstract

This paper presents an analysis of selected mathematics textbooks for the first level of primary schools. The aim of our study was to pinpoint tasks within these textbooks that contribute to the development of logical thinking in pupils. Based on a quantitative evaluation of these tasks, we proposed a task categorisation, which will serve as a basis for further research.

Keywords: logical thinking, tasks, mathematics textbooks, textbook analysis.

1. Introduction

This paper represents one of the outputs of the SGS 21374 Project: "Mathematics as an effective tool for developing logical thinking in primary school pupils". The aims of this project are to develop a set of activating tasks that will stimulate the development of pupils' logical thinking with the focus on formal logic, and to verify its use in practice. In order to achieve these aims, we carried out an analysis of selected mathematics textbooks for the first level of primary school. This analysis will then serve as a basis for selection and the development of tasks that are under-represented in the textbooks.

One of the goals of mathematics education at primary and secondary level is to develop pupils' logical thinking. This and other goals are set out in the Framework educational program for basic education (FEP BE, 2007). Despite the commonly accepted belief that mathematics contributes to the development of logical thinking, achieving this goal in practice isn't easy. This showed in the PIRLS & TIMSS international comparative studies of mathematics achievement among pupils in the Czech Republic (PIRLS & TIMSS, 2011).

Mathematics and mathematics education are based on a precise logical construction. However, passing this concept on to pupils in the classroom hasn't been successful, in spite of logical thinking being absolutely essential to understand mathematics. Without the right approach towards the development of logical thinking, the pupils' learned skills are only formal and short-term with no to little ability to make connections between knowledge. That is why we believe that the development of logical thinking is very important, particularly at the first level of primary school. Hence our analysis was focused on mathematics textbooks for the first level of primary schools.

The aim of our analysis was to determine the representation of tasks that stimulate the development of logical thinking. The selection of textbooks for the analysis was based on their use in teaching mathematics in Czech primary schools. In the next step we proposed a categorisation of these tasks.

2. Theoretical background

The concept of logical thinking has been studied and defined by a number of authors. To describe logical thinking, in his work Carroll (1972) used an example of a couple having a conversation. At the end of the conversation one of them said "That's quite logical." by which they meant that their statement irrevocably resulted from what had already been said and proven to be right. So, by Carroll definition, logic means a certain process of thinking, the ability to think correctly or, more precisely, the ability to reason, i.e., to draw conclusions from specific knowledge or thoughts (Carroll 1972).

According to Rice (2015), logical reasoning is all about being able to explain why something happens using facts and knowledge that we know to be true. Thinking logically enables us to effectively make decisions and predictions, and also to analyse and understand events that have already happened. In their studies, Artino (2008) and Widodo (2017) illustrated that the ability to think logically is a thinking process that uses mathematical reason and logic consistently so that an expected conclusion is obtained. This ability is needed not only to make good and just strategies but also in solving problems in daily life.

Piaget (1929, 1950) and his associates conducted systematic studies on the development of logical reasoning. In his theory, Piaget mentions that a person goes through several stages of cognitive development:

1. **The sensorimotor stage** occurs between 0-2 years of age, when they adapt to the surrounding environment and try to understand what they feel.
2. **The pre-operational stage** occurs between the ages of 2 and 7, when they can reason and practice their logical abilities. They use objects and symbols to represent something in a concrete form.
3. **The concrete operational stage** occurs between the ages of 7 and 11, when they use a logical reasoning process.
4. **The formal operational stage** is the last stage of the cognitive development and occurs from the age of 11 to adulthood, when their thinking is not only abstract but also becomes logical.

The ability to think logically is a characteristic of the last two stages of the cognitive developmental theory created by Piaget (1928). That is why our research focuses on the development of pupils' logical reasoning early at the first level of primary school education. Primary school is a stage of formal education that can determine how a child's characteristics develop. Pupils' skills start to develop at this stage, from their cognitive skills to their effective skills, and this affects their future thinking (Ristiana, 2019). If someone has good logical thinking skills, then they can resolve their problems well. This ability to solve problems ought to be developed early when they are in their "golden age", specifically at the primary school stage. However, it isn't easy to develop one's logical thinking skills. Many factors can influence the formation of one's logical thinking; the teacher being amongst the most influential (Purnami, 2018).

One of the goals of mathematics education is to develop the pupils' ability to solve simple practical tasks and problems by using logical thinking, rather than just applying common mathematical methods and algorithms. Solving logical problems that are in congruence with one's intellectual ability, can strengthen the pupils' ability to use logical thinking, even in those who usually don't excel at maths (RVP, 2007). Mathematics textbooks are an integral part of mathematics education that greatly contributes to the development of these key competencies (Molnar, 2007).

3. Logic at first level of primary school

The mathematics curriculum for the first level of primary school covers elements of logic that can be demonstrated in accurate expression, evaluation of judgements, and selecting the correct solution procedures. From the very start, pupils are taught how to make decisions about what is true and what is false. When teaching mathematics to pupils, we use the term "proposition" and decide on whether propositions are true or false. We also look at sentences as linguistic expressions of ideas. Thus, it is clear that logical thinking is closely associated with mother tongue and the ability to use sentences to express oneself (Melichar, 2007).

Teachers and pupils use logic operators, negations, quantifiers, and simple judgements in class; giving instructions and solving problems. Pupils tend to use the language of logic rather intuitively and often make mistakes when making judgements. This happens because they don't really understand the formality of logic. There are a number of studies that were dealing with the pupils' mistakes when using logic reasoning (Hoyles & Küchemann 2002, Stephanou & Pitta-Pantazi 2006).

There are two irreplaceable components in the development of pupils' logical thinking: the teacher and the textbook. Textbooks contain definitions, mathematical sentences and mathematical tasks/problems for pupils to solve. Practical experience shows that teachers use mathematics textbooks quite often as resources of both theoretical background and practical tasks.

4. Textbook analysis

The textbook analysis was focused on mathematics textbooks for the first level of primary school, and it was designed to detect the proportional representation of tasks that have a potential to stimulate the development of logical thinking. Our research focused solely on tasks that involved logic operators (conjunction, disjunction, implication, equivalence), negations, and quantifiers. We were interested in finding out how often these logic operators were involved in tasks and how often pupils come across their formal use. We believe that without a proper understanding of the basic principles of formal logic, the pupils' understanding of mathematics in general becomes distorted. Our analysis included textbooks from four different publishers. Considering the different stages of cognitive development (Piaget, 1928), we focused on textbooks for the 3rd and 5th year of primary school. Textbooks included in the analysis are shown in Table 1.

For each textbook, the total number of tasks was determined. Then, the number of tasks that involved logic operators, negations, and quantifiers was determined. From the data obtained, their proportional representation in each textbook was calculated.

Table 1. Textbooks used in this study

Title	**Author**	**Publisher**
Matematika 3: učebnice pro základní školy [Maths 3: Textbook for Primary Schools]	Hejný, M. et al.	Fraus
Matematika 5: učebnice pro základní školy [Maths 5: Textbook for Primary Schools]	Hejný, M. et al.	Fraus
Matematika: učebnice pro 3. ročník základní školy [Maths: Textbook for 3rd Year of Primary School]	Blažková, J. et al.	Didaktis
Matematika: učebnice pro 5. ročník základní školy [Maths: Textbook for 5th Year of Primary School]	Blažková, J. et al.	Didaktis
Matematika pro 3. ročník základních škol [Maths for 3rd Year of Primary School]	Blažková, R. et al.	Alter
Matematika pro 5. ročník základních škol [Maths for 5th Year of Primary School]	Justová, J.	Alter
Matematika pro 3. ročník základní školy [Maths for 3rd Year of Primary School]	Čížková, M.	SPN
Matematika pro 5. ročník základní školy [Maths for 5th Year of Primary School]	Vacková, I. et al.	SPN

Based on the analysis, for the purpose of our research we proposed defined categories of tasks that involved logic operators. The tasks were divided into 6 categories: L1 - L6, in regards to their appearance in the textbooks. When creating these categories, the following were taken into consideration: the context, type, and difficulty of a task. We also considered the way a task was presented, and the way pupils have to work with the given information. We believe that it is more difficult to correctly interpret tasks where logic operators are used, than to simply follow a set of instructions. The proposed categories of tasks are shown in Table 2. All types of these tasks involve some form of a logical operator, negation, or quantifiers and their proper understanding is crucial for the correct solution. When presented in this way, tasks can contribute to the formation of pupils' logical thinking and the development of the basic elements of formal logic. The examples of tasks are not their exact quotations, though they are inspired by real tasks in the textbooks.

Table 2. Categories of logical tasks in mathematics textbooks

Category Type	**Category Name**	**Category Description**
L1	Explicit	Tasks involving a conjunction in terms of formal logic, i.e. a conjunction of two various propositions. For example: • *"A chocolate bar costs 15 CZK and they are sold in two's. Write down the price of 2, 4, …, 20 bars."* • *"There were 6 stones in a pile. Jana and Katka took turns removing one or two stones. The person who removed the last stone won. Who's the winner?"* • *"Can you guess the number I'm thinking of? If I multiply my number by 8, I get 984."*
L2	Series Of Elements	Tasks involving a conjunction in terms of a series of elements that carry additional information. For example: • *"My aunt was making marmalade. She used 300g of cherries, 400g of peaches, and 500g of apples. How many kilograms of fruit did she use altogether?"* • *"A sports club bought 13 volleyballs for 550 CZK each, three nets for 3700 CZK each, and 25 handballs for 690 CZK each. How much money did they spend on the equipment?"* • *"Three friends had a race on their bicycles. Petr finished the race in 1 hour and 5 minutes. Honza took 54 minutes to finish the race, and Karel finished it in 3600 seconds. Who came in first/second/third?*"
L3	Instructions	Tasks involving a conjunction in terms of a series of instructions to follow. For example: • *"Calculate and put results in the correct order."* • *"Calculate and perform a test of results."* • *"Perform column addition and place results on the number line."* Technically, these are not propositions, but a proper understanding of these instructions promotes a proper understanding of the concept of conjunctions.
L4	Quantifier	Tasks involving any kind of a quantifier. For example: • *"There are 22 pieces of chocolate in the box. Each piece weighs 300g. How much does the whole box weigh?"* • *"Take the numbers in the box and reduce them all by 5."* • *"In your notebook, draw three concurrent lines and mark their points of intersection."*
L5	Composite Proposition	Tasks involving a conjunction in terms of a connection between two propositions. A proper understanding of the composite proposition is crucial in order to solve the problem. For example: • *"Jan brought 56 candies to birthday party. There were 8 children at the birthday party. How many candies did each one of them get, if all treats were given away and all children were given the same number of candies?"* • *"What is the dividend, if the divisor is 4 and the quotient equals 3497284?"*

L6	Logical Tasks	Tasks that can be regarded as "recreational mathematical tasks". Such tasks have a great potential to promote logical thinking. In order to solve them, it is essential to draw conclusions from given information by using both linguistic and mathematical knowledge. For example: • *"I'm thinking of a number. My friend is thinking of a number that is larger than mine by 6. Now take his number, add 3 and you will get 39. What number was I thinking of?"* • *"Carry on the number series: 102; 204; 306; 408; 510; …"* • *"Determine where the Novak, Becka, Soucek, Safranek, and Ptacek families live if you know, that:* *The Soucek family live 2 floors below the Novak family.* *The Becka family live neither on the bottom nor the top floor.* *The Novak family live on the second top floor.* *The Safranek family don't live on the top floor."* • *Tasks involving knights and knaves.*

5. Results and discussion

The representation of logical tasks in individual mathematics textbooks is illustrated in Graph 1. Looking at the graph, it is obvious that the number of logical tasks in a textbook differs from author to author. The mean proportional representation of studied tasks was 36% and the mean number of tasks in one textbook was 532. That means that, on average, there were 192 tasks involving logical operators, negations, and/or quantifiers in each textbook. These results seem promising.

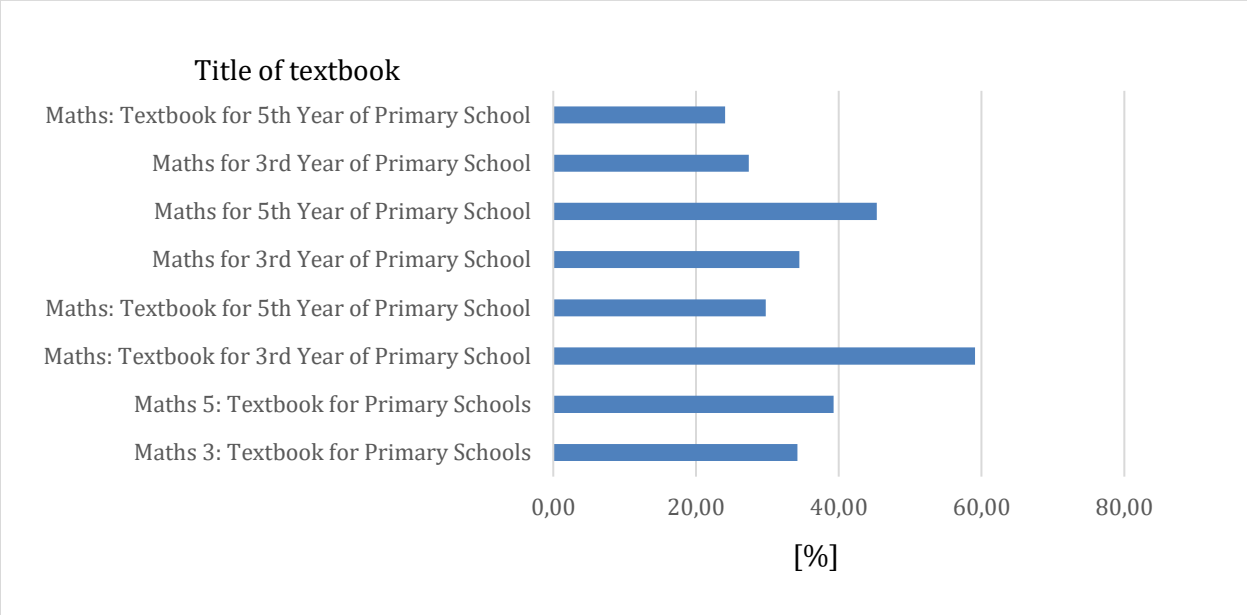

Chart 1. Proportional representation of studied tasks in mathematics textbooks for 3rd and 5th years of primary school

The proportional representation of logical tasks in mathematics textbooks is better illustrated in Graph 2. The graph shows proportional representation of the individual categories of tasks (L1-L6).

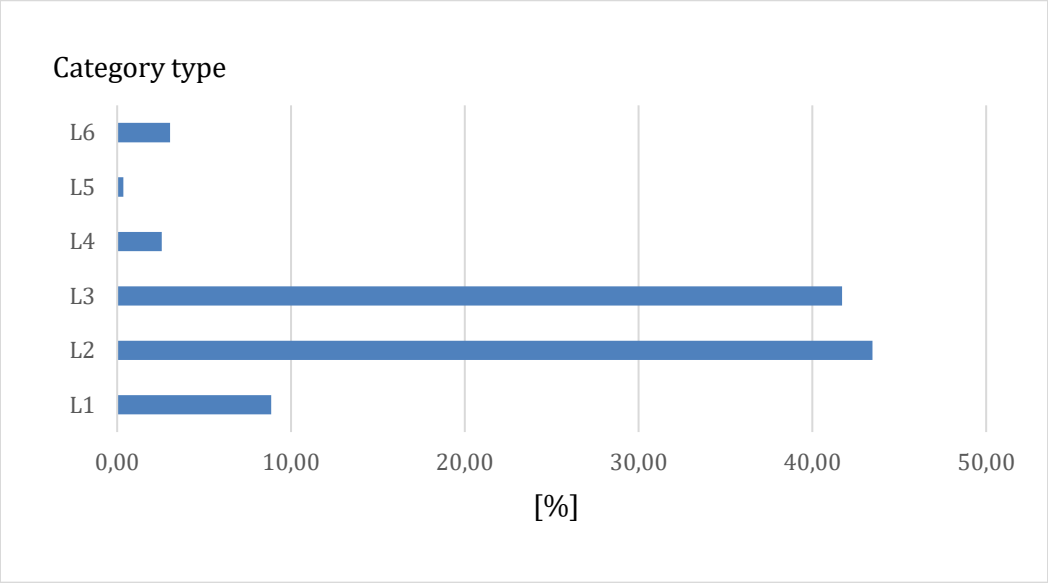

Chart 2. Proportional representation of categories of logical tasks in mathematics textbooks

The results clearly show that there were 2 categories of tasks with high proportional representation: L2 (Series of Elements) with 43% representation and L3 (Instructions) with 42% representation. With regard to 1st level mathematics, these results seem promising, however, we must realize that tasks in these categories involved nothing but conjunctions. Moreover, in the case of category L3, these conjunctions weren't even logic operators, they were simply a series of instructions, e.g. *"Calculate and put results in the correct order."*. In terms of logic, these instructions are not propositions, although they can promote a proper understanding of logic conjunctions. Category L1 (Explicit) had the third highest representation of nearly 9%. This category involved all logic operators, but mostly implications, e.g. *"How many litres of petrol were consumed if the average fuel consumption of a car was 7 litres per 100 kilometres?"*. Note that this example is a special case of implication with a true premise. Although it is okay to use this type of implication in this context, it raises a question of whether it might lead to misconception of the implication as it may be mistaken for equivalence. Besides the implications, there were only a few conjunctions and disjunctions involved in this category (L1), and no equivalence nor negations were involved whatsoever.

Category L6 (Logical Tasks) had a representation of 3%. This is a category that is worthy of note as it has a great potential in terms of the development of logical thinking. It included so-called "recreational mathematical tasks" and tasks involving explicit logic operators. In order to solve these tasks, one must come to conclusions by applying their prior knowledge, skills, and experience. For example, *"If x and y are even, then x + y is even. Is this statement true?"* (Maths 5:

Textbook for Primary Schools, Hejny et al., 2001). It is a shame that these tasks were included only in three of the studied mathematics textbooks: the two textbooks published by Fraus Publishing, and the textbook for 3rd year mathematics published by SPN.

This relatively small representation of tasks involving explicit logic operators that promote the development of logical thinking, might be one of the reasons why the success rate of Czech pupils in the 2011 TIMSS study was so low (PIRLS & TIMSS, 2011). The results of the 2011 TIMSS study showed that only a small minority of pupils got average scores in both mathematics and reading literacy (PIRLS & TIMSS, 2011).

6. Conclusion

This research involved an analysis of selected mathematics textbooks for the first level of primary school. The analysis was focused on tasks that had the potential to facilitate the development of logical thinking in pupils. We were particularly interested in tasks that involved logic operators (conjunctions, disjunctions, implications, equivalences), negations, and quantifiers. Based on this analysis, we proposed 6 categories of tasks (see Table 2) that involved logic operators of some sort. On average, tasks involving logic operators were represented in the textbooks by 34%. However, 85% of these were tasks in categories L2 and L3, which involved nothing but conjunctions. Furthermore, in half of these cases the conjunctions involved weren't even logic operators, but they were simply a series of instructions. Yet, we see the biggest problem in the very low representation of tasks from the remaining categories, as they were represented only by 15% altogether. That doesn't leave much space for neither the pupils nor the teachers to develop the pupils' logical thinking. With that being said, note that the teachers' role in the development of pupils' logical thinking is vital.

This analysis of mathematics textbooks created space for us in future research, to focus on creating activities that involve tasks, which are currently under-represented in the textbooks. We aim to focus on activities involving explicit use of logic operators, negations, and quantifiers.

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