

## **CHESS PIECE TOUR PROBLEMS**

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### **Abstract**

Chess and other board games can significantly increase the level of mathematical literacy of pupils aged 6 to 11. Pupils can play chess, solve chess diagrams or deal with mathematical chess problems as for example independence problems or dominance problems. In our paper, we will focus on chess piece tour problems. We will reduce the classic chessboard  $8 \times 8$  to a smaller  $5 \times 5$ ,  $6 \times 6$  and  $7 \times 7$  chessboard, respectively, to make chess piece tour problems more accessible to pupils aged 6 to 11.

**Keywords:** Mathematical chess problems, combinatorics, Mathematical Kangaroo.

### **1. Introduction**

In our paper we will deal with mathematical chess problems, especially with chess piece tour problems. We recall (“Mathematical chess problems”, Wikipedia) which is formulated using a chessboard and chess pieces.

Chess piece tour problems ask to find a tour of certain chess piece, which visits all squares on a chess board (“Mathematical chess problems”, Wikipedia). It is very easy to find such tour for king, queen and rook. Since bishops are able to reach only squares of one color, the problem is formulated for one color. The most interesting is finding a tour for knight, i.e. Knight's\_Tour.

The previous problems have been resolved generally for the chessboard  $m \times n$ , but we will focus on a smaller  $5 \times 5$ ,  $6 \times 6$  and  $7 \times 7$  chessboard, respectively, in order to make chess independence problems more accessible to pupils aged 6 to 11.

The reader can recall the movements of chess pieces (king, queen, rook, bishop and knight) using e.g. (“Rules of chess”, Wikipedia). We only show the movement of a special piece named kangaroo that was introduced in Mathematical Kangaroo (“Matematický klokan 2015”). A kangaroo moves three squares horizontally then one square vertically, or one square horizontally then three squares vertically, see Figure 1.

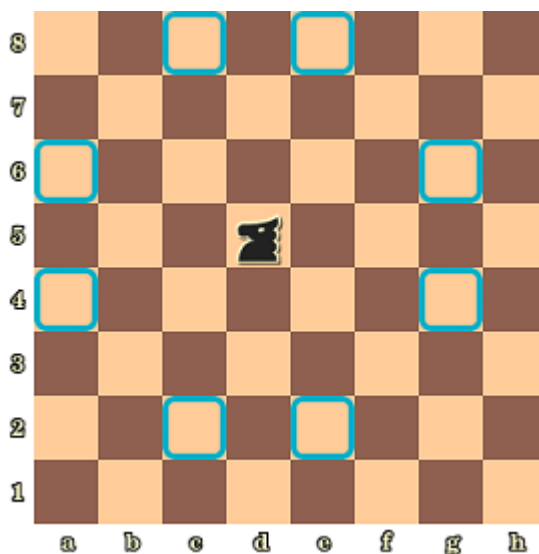


Figure 1. Moves of a kangaroo

## 2. Tour for queen, rook and king

Finding a tour for a queen is very easy as we can see on Figures 2-7. We can use different strategies in tours and we can also choose the different starting field (the starting field is highlighted by red colour and the ending field is highlighted by blue colour). We notice that the tours presented on Figures 2-4 can be used also for a rook or a king and the tours presented on Figures 5-7 can be used also for a king. In the given examples, the queen always moves by only one square. The queen could, of course, move more squares (such as the bishop in the next section).

<b>1</b>	2	3	4	5
10	9	8	7	6
11	12	13	14	15
20	19	18	17	16
21	22	23	24	<b>25</b>

Figure 2. Tour for a queen (rook, king) on 5x5

<b>36</b>	<b>35</b>	<b>34</b>	<b>33</b>	<b>32</b>	<b>31</b>
<b>9</b>	<b>10</b>	<b>11</b>	<b>28</b>	<b>29</b>	<b>30</b>
<b>8</b>	<b>7</b>	<b>12</b>	<b>27</b>	<b>26</b>	<b>25</b>
<b>5</b>	<b>6</b>	<b>13</b>	<b>22</b>	<b>23</b>	<b>24</b>
<b>4</b>	<b>3</b>	<b>14</b>	<b>21</b>	<b>20</b>	<b>19</b>
<b>1</b>	<b>2</b>	<b>15</b>	<b>16</b>	<b>17</b>	<b>18</b>

Figure 3. Tour for a queen (rook, king) on 6×6

<b>31</b>	<b>30</b>	<b>29</b>	<b>28</b>	<b>27</b>	<b>26</b>	<b>49</b>
<b>32</b>	<b>13</b>	<b>12</b>	<b>11</b>	<b>10</b>	<b>25</b>	<b>48</b>
<b>33</b>	<b>14</b>	<b>3</b>	<b>2</b>	<b>9</b>	<b>24</b>	<b>47</b>
<b>34</b>	<b>15</b>	<b>4</b>	<b>1</b>	<b>8</b>	<b>23</b>	<b>46</b>
<b>35</b>	<b>16</b>	<b>5</b>	<b>6</b>	<b>7</b>	<b>22</b>	<b>45</b>
<b>36</b>	<b>17</b>	<b>18</b>	<b>19</b>	<b>20</b>	<b>21</b>	<b>44</b>
<b>37</b>	<b>38</b>	<b>39</b>	<b>40</b>	<b>41</b>	<b>42</b>	<b>43</b>

Figure 4. Tour for a queen (rook, king) on 7×7

<b>1</b>	<b>23</b>	<b>3</b>	<b>25</b>	<b>5</b>
<b>22</b>	<b>2</b>	<b>24</b>	<b>4</b>	<b>6</b>
<b>21</b>	<b>20</b>	<b>12</b>	<b>13</b>	<b>7</b>
<b>19</b>	<b>17</b>	<b>14</b>	<b>11</b>	<b>8</b>
<b>18</b>	<b>15</b>	<b>16</b>	<b>10</b>	<b>9</b>

Figure 5. Tour for a queen (king) on 5×5

21	22	30	31	35	36
11	20	23	29	32	34
10	12	19	24	28	33
4	9	13	18	25	27
3	5	8	14	17	26
1	2	6	7	15	16

Figure 6. Tour for a queen (king) on 6×6

7	8	9	10	11	12	13
34	6	24	23	17	16	14
35	33	5	25	22	18	15
36	46	32	4	26	21	19
37	47	45	31	3	27	20
38	48	49	44	30	2	28
39	40	41	42	43	29	1

Figure 7. Tour for a queen (king) on 7×7

### 3. Tour for bishop

We recall that the tour problem for a bishop is formulated for one color. Finding the tour is not very difficult, see Figures 8-10.

5		7		2
	6		8	
12		3		9
	13		10	
1		11		4

Figure 8. Tour for a bishop on 5×5

	<b>6</b>		<b>11</b>		<b>2</b>
<b>17</b>		<b>7</b>		<b>12</b>	
	<b>18</b>		<b>8</b>		<b>10</b>
<b>14</b>		<b>3</b>		<b>9</b>	
	<b>13</b>		<b>16</b>		<b>5</b>
<b>1</b>		<b>15</b>		<b>4</b>	

Figure 9. Tour for a bishop on 6×6

<b>12</b>		<b>6</b>		<b>15</b>		<b>2</b>
	<b>10</b>		<b>17</b>		<b>24</b>	
<b>9</b>		<b>14</b>		<b>5</b>		<b>23</b>
	<b>18</b>		<b>1</b>		<b>20</b>	
<b>16</b>		<b>4</b>		<b>22</b>		<b>7</b>
	<b>25</b>		<b>19</b>		<b>13</b>	
<b>3</b>		<b>21</b>		<b>8</b>		<b>11</b>

Figure 10. Tour for a bishop on 7×7

#### 4. Tour for knight

Finding a tour for a knight is more complicated than the previously mentioned chess pieces. Figures 11-13 illustrate possible solutions for chessboards 5×5, 6×6 and 7×7, respectively.

<b>5</b>	<b>16</b>	<b>21</b>	<b>10</b>	<b>3</b>
<b>22</b>	<b>11</b>	<b>4</b>	<b>15</b>	<b>20</b>
<b>17</b>	<b>6</b>	<b>23</b>	<b>2</b>	<b>9</b>
<b>12</b>	<b>25</b>	<b>8</b>	<b>19</b>	<b>14</b>
<b>7</b>	<b>18</b>	<b>13</b>	<b>24</b>	<b>1</b>

Figure 11. Knight's\_Tour on 5×5

<b>1</b>	16	9	32	3	18
26	31	2	17	10	33
15	8	27	34	19	4
28	25	30	11	22	35
7	14	23	20	5	12
24	29	6	13	<b>36</b>	21

Figure 12. Knight's\_Tour on 6×6

<b>1</b>	12	23	46	3	14	25
22	<b>49</b>	2	13	24	35	4
11	28	45	40	47	26	15
42	21	48	27	34	5	36
29	10	41	44	39	16	33
20	43	8	31	18	37	6
9	30	19	38	7	32	17

Figure 13. Knight's\_Tour on 7×7

### 5. Tour for kangaroo

With respect to the movement of a kangaroo, the tour problem for this chess piece is formulated only for one color as in the case of bishop, see Figures 14-16.

	5		3	
8		10		<b>12</b>
	<b>1</b>		7	
6		4		2
	9		11	

Figure 14. Tour for a kangaroo on 5×5

	14		6		2
5		3		13	
	10		16		18
15		7		1	
	4		12		8
11		9		17	

Figure 15. Tour for a kangaroo on 6×6

	14		18		6	
19		7		15		17
	24		20		10	
13		11		5		21
	8		2		16	
1		23		9		3
	12		4		22	

Figure 16. Tour for a kangaroo on 7×7

## 6. Conclusion

We were interested in some chess piece tour problems that can be solved by pupils aged 6 to 11. A special attention was focused on a special piece named kangaroo.

To solve the previous problems pupils can use a squared paper or they can use some computer application, see e.g. (“Chess Diagram Setup”).

Some of the previous problems are very simple (such as the problem with Figure 2) and pupils can solve them very quickly. Some problems may be more difficult to some pupils aged 6-11 (such as the problem associated with Figure 13). In this case, pupils can try to travel the figure as long as possible. Solving the previous problems may be preceded by solving some preparatory tasks, such as the task of getting a chess piece from one chessboard field to another.

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