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CHESS INDEPENDENCE PROBLEMS

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Abstract

Board games can significantly develop combinatorial skills (not only) of pupils aged 6 to 11. Chess is one of the oldest and most famous games. In addition to playing chess games and practicing chess situations (for example solving diagrams) there are some mathematical chess problems that can help to develop combinatorial skills. In our paper we will focus on independence problems. We will reduce the classic chessboard 8×8 to a smaller 4×4 and 6×6 chessboard, respectively, to make chess independence problems more accessible to pupils aged 6 to 11.

Keywords: Mathematical chess problems, independence problems, Mathematical Kangaroo

1. Introduction

First of all, we would like to introduce to readers what mathematical chess problems mean, especially independence problems. We will explain these concepts by means of ("Mathematical chess problems", Wikipedia).

A mathematical chess problem is a mathematical problem that is formulated using a chessboard and chess pieces. Many very good mathematicians, such as Leonhard Euler and Carl Friedrich Gauss, have studied mathematical chess problems.

Independence problems are a family of the following problems. Given a certain chess piece (queen, rook, bishop, knight, or king) find the maximum number of such pieces, which can be placed on a chessboard so that none of the pieces attacks each other. It is also required that an actual arrangement for this maximum number of pieces be found.

The previous problems have been resolved generally for the chessboard n×n (Gik, 2019), but we will reduce the classic chessboard 8×8 to a smaller 4×4 and 6×6 chessboard, respectively, in order to make chess independence problems more accessible to pupils aged 6 to 11.

We will recall ("Rules of chess", Wikipedia) the movements of chess pieces (king, queen, rook, bishop and knight), and the movement of a special piece named kangaroo that was introduced in Mathematical Kangaroo ("Matematický klokan 2015").

a) King
A king moves exactly one square horizontally, vertically, or diagonally, see Fig. 1.

b) Queen
A queen moves any number of vacant squares in a horizontal, vertical, or diagonal direction, see Fig. 2.

c) Rook
A rook moves any number of vacant squares in a horizontal or vertical direction, see Fig. 3.

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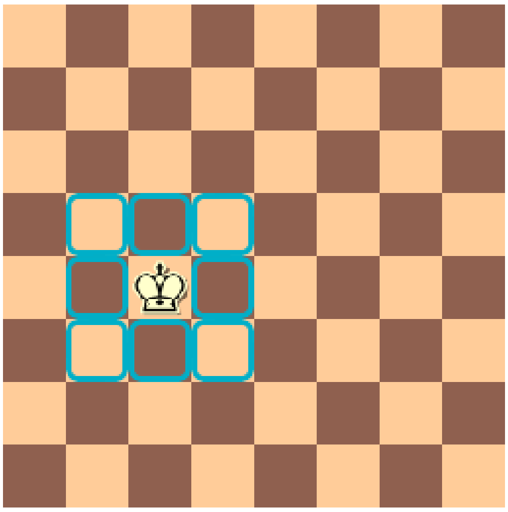

Figure 1. Moves of a king

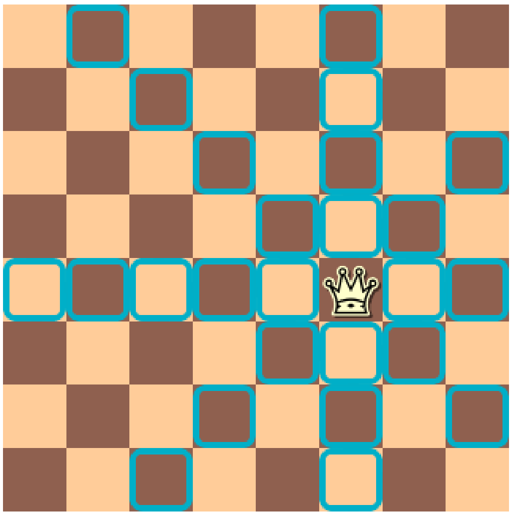

Figure 2. Moves of a queen

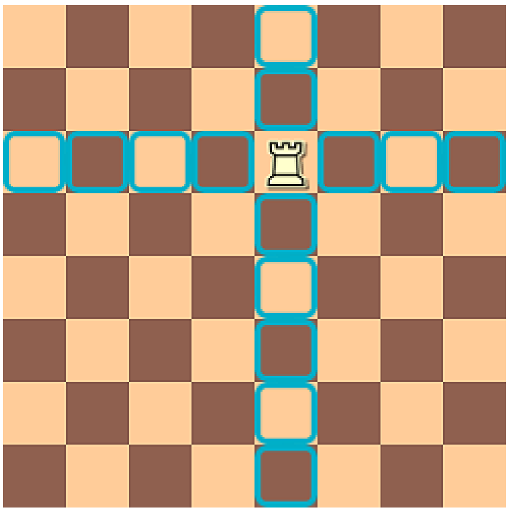

Figure 3. Moves of a rook

d) Bishop

A bishop moves any number of vacant squares in any diagonal direction, see Fig. 4.

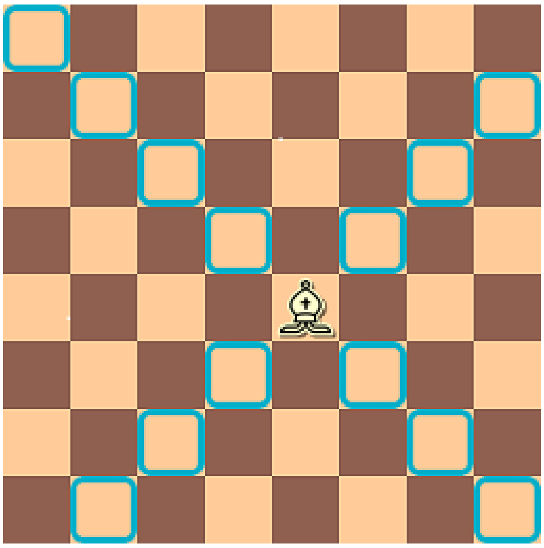

Figure 4. Moves of a bishop

e) Knight

A knight moves two squares horizontally then one square vertically, or one square horizontally then two squares vertically, see Fig. 5.

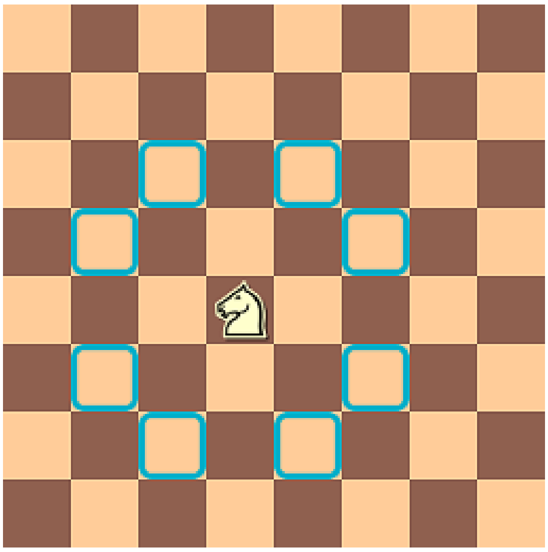

Figure 5. Moves of a knight

f) Kangaroo

A kangaroo moves three squares horizontally then one square vertically, or one square horizontally then three squares vertically, see Fig. 5.

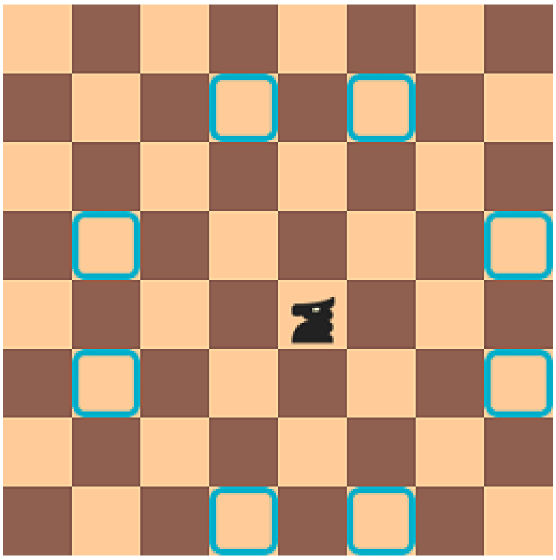

Figure 6. Moves of a kangaroo

2. 4×4 and 6×6 chessboards

Starting this section, we will recall (Gik, 2019) the general results of independence problems for the n×n chessboard and chess pieces and show examples for the 4×4 and 6×6 chessboards.

A. King

Since the maximum number of kings, which can be placed on a n×n chessboard so that none of the kings attacks each other is k^2, where $n = 2 \cdot k$ (if n is even) or $n = 2 \cdot k - 1$ (if n is odd), we have that the maximum number for the 4×4 chessboard equals 4, and the maximum number for the 6×6 chessboard equals 9. See Fig. 7 and 8, respectively.

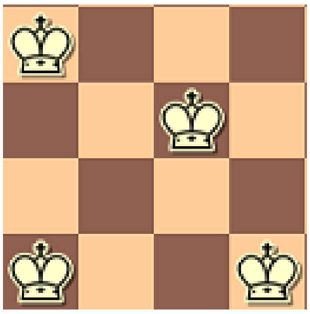

Figure 7. 4 kings

B. Queen

Since the maximum number of queens, which can be placed on a n×n chessboard so that none of the queens attacks each other is n for $n \geq 4$, we have that the maximum number for the 4×4 chessboard equals 4, and the maximum number for the 6×6 chessboard equals 6. See Fig. 9 and 10, respectively.

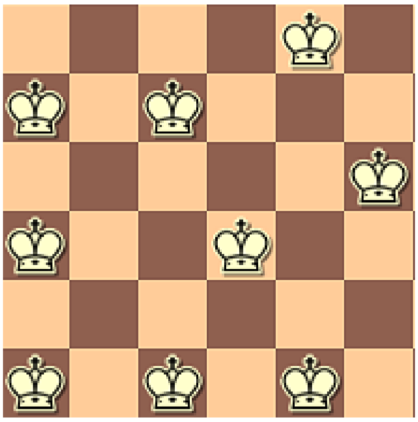

Figure 8. 9 kings

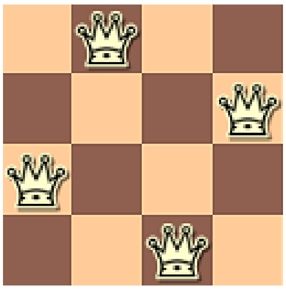

Figure 9. 4 queens

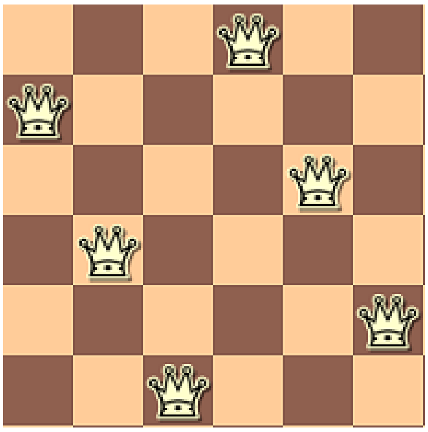

Figure 10. 6 queens

C. Rook

Since the maximum number of rooks, which can be placed on a n×n chessboard so that none of the rooks attacks each other is n, we have that the maximum number for the 4×4 chessboard equals 4, and the maximum number for the 6×6 chessboard equals 6. See Fig. 11 and 12, respectively.

D. Bishop

Since the maximum number of bishops, which can be placed on a n×n chessboard so that none of the bishops attacks each other is 2n-2 for n ≥ 2, we have that the maximum number for the 4×4 chessboard equals 6, and the maximum number for the 6×6 chessboard equals 10. See Fig. 13 and 14, respectively.

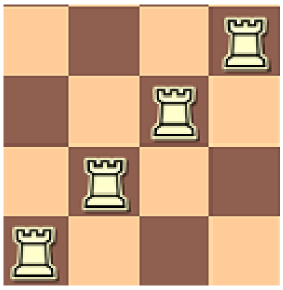

Figure 11. 4 rooks

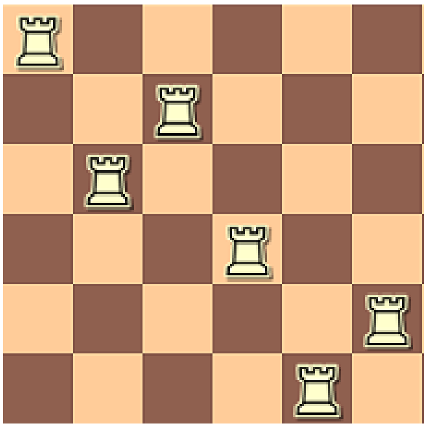

Figure 12. 6 rooks

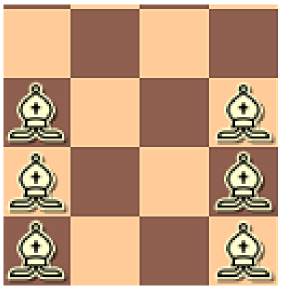

Figure 13. 6 bishops

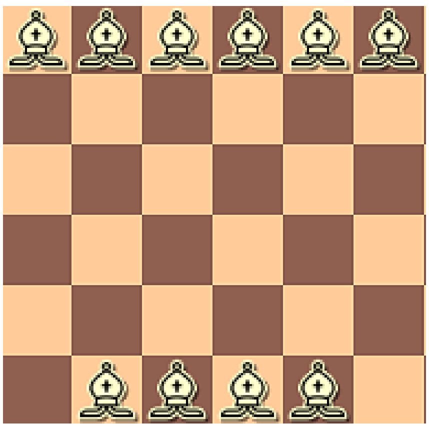

Figure 14. 10 bishops

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E. Knight

Since the maximum number of knights, which can be placed on a n×n chessboard so that none of the knights attacks each other is $n^2/2$ whenever n is even, and $(n^2+1)/2$ whenever n is odd, we have that the maximum number for the 4×4 chessboard equals 8, and the maximum number for the 6×6 chessboard equals 18. See Fig. 15 and 16, respectively.

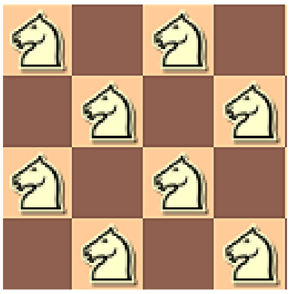

Figure 15. 8 knights

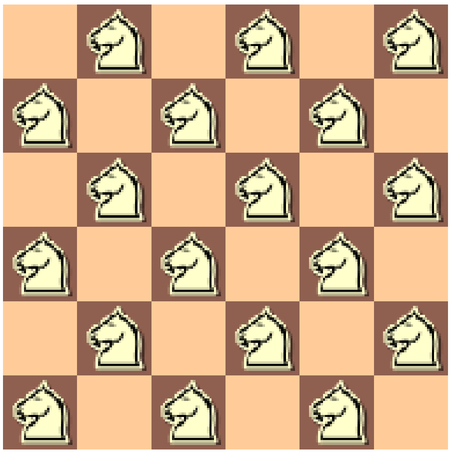

Figure 16. 18 knights

Finishing this section, we will solve the independence problem for the kangaroo and the 4×4 and 6×6 chessboards.

i) 4×4 chessboards

We can notice that a kangaroo on an interior square of 4x4 chessboard does not attack any square, so we can place kangaroos on the all four interior squares of 4x4 chessboard. Since a kangaroo on an outer square of 4x4 chessboard attacks just two squares, it follows that we can occupy by kangaroos maximally one half of the outer squares of 4x4 chess board, i.e. six squares. Figure 17 illustrates that the number 6 is possible in fact, thus the maximum number of kangaroos, which can be placed on 4x4 chessboard so that none of the kangaroos attack each other is 10.

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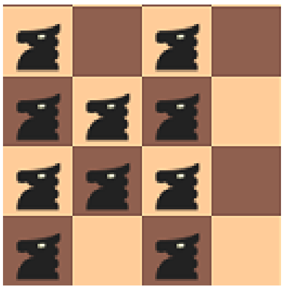

Figure 17. 10 kangaroos

ii) 6×6 chessboards

A kangaroo on any square of 6x6 chessboard attacks at least 2 squares. Thus we can occupy by kangaroos maximally one half of all squares of 6x6 chessboard, i.e. 18 squares. Figure 18 illustrates that the number 18 is possible in fact.

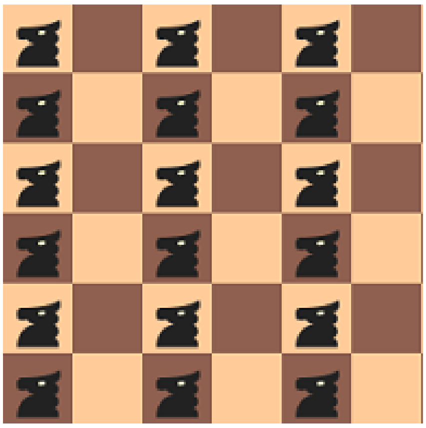

Figure 18. 18 kangaroos

3. Conclusion

We were interested in some chess independence problems that can be solved by pupils aged 6 to 11. A special attention was focused on a special piece named kangaroo.

To solve the previous problems pupils can use, for example, a printed chessboard and beans instead of pieces, or they can use some computer application, see e.g. ("Chess Diagram Setup").

Another kind of mathematical chess problems is a covering problem. In these problems it is requested to find a minimum number of pieces of the given kind and place them on a chess board in such a way, that all free squares of the board are attacked by at least one piece ("Mathematical chess problems", Wikipedia).

Acknowledgements

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